Introduction

Let $G$ be a finite group and $p$ a prime. Let $(K, R, k)$ be a $p$-modular system. Let $(\pi)$ be the maximal ideal of $R$. We assume that $K$ contains the $|G|$-th roots of unity and that $k$ is algebraically closed. Let $\nu$ be the valuation of $K$ normalized so that $\nu(p)=1$. For an $(R$-free) $RG$-module $U$ lying in a block $B$ of $G$, we define $ht(U)$, the height of $U$, by $ht(U) = \nu(\text{rank}_R U) - \nu(|G|) + d(B)$, where $d(B)$ is the defect of $B$. The heights of $kG$-modules are defined in a similar way, and heights are always nonnegative. In this paper we study indecomposable $RG$-(or $kG$-) modules of height zero, especially their behaviors under the block induction. In section 1 we introduce, motivated by Broué [7], the notion of linkage for arbitrary block pairs as a generalization of the one for Brauer pairs, and establish fundamental properties about it. In section 2 we give a condition for a block of a normal subgroup to be induced to the whole group. In section 3 a characterization of $RG$-(or $kG$-) modules of height 0 via their vertices and sources is given, which generalizes a result of Knörr [14]. Based on this result it is shown in section 4 that for any irreducible character $\chi$ of height 0 in $B$ and any normal subgroup $N$ of $G$, $\chi_N$ contains an irreducible character of height 0. This is well-known when $B$ is weakly regular with respect to $N$. An answer to the problem of determining which irreducible (Brauer) characters of $N$ appear as irreducible constituents of irreducible (Brauer) characters of height 0 is also obtained (Theorem 4.4). In section 5 a generalization of a theorem of Isaacs and Smith [11] is given. In section 6 an alternative proof of a theorem of Berger and Knörr [1] is given.

1. Block induction and characters of height 0

Throughout this section $H$ is a subgroup of $G$, and $B$ and $b$ are $p$-blocks of $G$ and $H$, respectively.

Let $G_p$ be the set of $p$-regular elements of $G$, $ZRG$ the center of $RG$, and $ZRG_p$ be the $R$-submodule of $ZRG$ spanned by $p$-regular conjugacy class sums.