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BLOCK INDUCTION, NORMAL SUBGROUPS AND CHARACTERS OF HEIGHT ZERO

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Introduction

Let G be a finite group and p a prime. Let (K, R, k) be a p-modular system. Let (π) be the maximal ideal of R. We assume that K contains the $|G|$-th roots of unity and that k is algebraically closed. Let ν be the valuation of K normalized so that $\nu(p)=1$. For an (R-free) RG-module U lying in a block B of G, we define $ht(U)$, the height of U, by $ht(U)=\nu(\mathrm{rank}_R U)-\nu(|G|)+d(B)$, where $d(B)$ is the defect of B. The heights of kG-modules are defined in a similar way, and heights are always nonnegative. In this paper we study indecomposable RG-(or kG-) modules of height zero, especially their behaviors under the block induction. In section 1 we introduce, motivated by Broué [7], the notion of linkage for arbitrary block pairs as a generalization of the one for Brauer pairs, and establish fundamental properties about it. In section 2 we give a condition for a block of a normal subgroup to be induced to the whole group. In section 3 a characterization of RG-(or kG-) modules of height 0 via their vertices and sources is given, which generalizes a result of Knörr [14]. Based on this result it is shown in section 4 that for any irreducible character χ of height 0 in B and any normal subgroup N of G, χ_N contains an irreducible character of height 0. This is well-known when B is weakly regular with respect to N. An answer to the problem of determining which irreducible (Brauer) characters of N appear as irreducible constituents of irreducible (Brauer) characters of height 0 is also obtained (Theorem 4.4). In section 5 a generalization of a theorem of Isaacs and Smith [11] is given. In section 6 an alternative proof of a theorem of Berger and Knörr [1] is given. Throughout this paper an RG-module is assumed to be R-free of finite rank.

1. Block induction and characters of height 0

Throughout this section H is a subgroup of G, and B and b are p-blocks of G and H, respectively.

Let $G_{p'}$ be the set of p-regular elements of G, ZRG the center of RG, and $ZRG_{p'}$ be the R-submodule of ZRG spanned by p-regular conjugacy class sums.