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ALMOST QF RINGS WITH $J^3=0$

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In this paper we always assume that R is a two-sided artinian ring with identity. In [3] we have defined right almost QF rings and showed that those rings coincided with rings satisfying $(*)^*$ in [2], which K. Oshiro [5] called co-H rings. We shall show in Section 2 that right almost QF rings are nothing but direct sums of serial rings and QF rings, provided $J^3=0$. Further in Section 5 we show that if R is a two-sided almost QF ring and $1=e_1+e_2+e_3$, then R has the above structure, provided $J^4=0$, where $\{e_i\}$ is a complete set of mutually orthogonal primitive idempotents. Moreover if $1=e_1+e_2+e_3+e_4$, we have the same result except one case. We shall study, in Section 3, right almost QF rings with homogeneous socles $W_k^n(Q)$ [7] and give certain conditions on the nilpotency m of the radical of $W_k^n(Q)$, under which $W_k^n(Q)$ is left almost QF or serial. In particular if $m\leqq 2n$, $W_k^n(Q)$ is serial. We observe a special type of almost QF rings such that every indecomposable projective is uniserial or injeative in Section 4.

1. Almost QF rings

In this paper we always assume that R is a two-sided artinian ring with identity and that every module M is a unitary right R-module. By $\bar{M}$ we denote $M/\mathrm{J}(M)$, where $\mathrm{J}(M)$ is the Jacobson radical of M. We use the same notations in [3]. We call R *a right almost* QF *ring* if R is right almost injective as a right R-module [3] and [4]. We can define similarly *a left almost* QF *ring.* If R is a two-sided almost QF ring, we call it simply *an almost* QF *ring.* It is clear that R is right almost QF if and only if every finitely generated projective R-modlue is right almost injective. Hence the concept of almost QF rings is preserved under Morita equivalence and we may assume that R is basic.

In this section we shall give some results which we use later. First we give a property of any right almost QF rings.

Proposition 1. *Assume that R is right almost* QF. *Let e_1R be injective, e_1J^i be projective, i.e., $e_1J^i\approx e_{\rho(i)}R$ for all $i\leqq$(some k) and $e_1J^{k+1}/e_1J^{k+2}\approx \bar{e}_a\bar{R}\oplus\cdots$.*