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ALMOST QF RINGS AND ALMOST QF$^{\#}$ RINGS

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In this paper we assume that every ring R is an associative ring with identity and R is two-sided artinian. The author has defined almost projective modules and almost injective modules in [8], and by making use of the concept of almost projectives he has defined almost hereditary rings in [7], whose class contains that of hereditary rings and serial rings. Similarly to [7] we shall define an almost QF ring, which is a generalization of QF rings.

It is well known that an artinian ring R is QF if and only if R is self injective. Following this fact, if R is almost injective as a right R-module, we call R a right almost QF ring. Analogously we call R a right almost QF$^{\#}$ ring if every injective is right almost projective. On the other hand, the author studied rings with (*) (resp. (*)*) (see §1 for definitions) in [4]. K. Oshiro called such a ring a right H- (resp. co-H) ring in [10]. In this note we shall show that a right almost QF (resp. almost QF$^{\#}$) ring coincides with a right co-H (resp. H-) ring. In the final section we shall give a characterization of serial rings in terms of almost projectives and almost injectives.

In the forthcoming paper [9] we shall study certain conditions under which right almost QF rings are QF or serial.

1. Almost QF rings

In this paper we always assume that R is a two-sided artinian ring with identity and that every module is a unitary right R-module. We use the same notations in [7]. We have studied almost hereditary rings in [7], i.e. J, the Jacobson radical of R, is right almost projective. We shall study, in this paper, some kind of the dual concept to almost hereditary rings (see Theorem 1 below). We call R *a right almost* QF *ring* if R is right almost injective as a right R-module [8]. We can define similarly a *left almost* QF *ring*. It is clear that R is right almost QF if and only if every finitely generated projective R-module is right almost injective. Hence the concept of almost QF rings is presevred under Morita equivalence and we may assume that R is basic.

On the other hand, the author studied the two conditions in [3] and [4]. Let M be an R-module. If $M\mathrm{Soc}^{l}(R)\neq 0$ (resp. $M\,\mathrm{Soc}^{r}(R)\neq 0$) then we call M non-