tom Dieck, T. and Petrie, T.
Osaka J. Math.
30 (1993), 855-886

HOMOLOGY PLANES AND ALGEBRAIC CURVES

TAMMO TOM DIECK AND TED PETRIE

(Received July 13, 1992)

1. Introduction and results

This paper is concerned with the construction of acyclic affine surfaces from plane algebraic curves.

Surfaces will always be connected, non-singular, quasi-projective, algebraic surfaces over the complex numbers. Let R be a subring of the rational numbers. A surface V is called *R-homology plane* if $H_i(V; R)=0$ for $i>0$. In the case $R=\boldsymbol{Z}$ we simply refer to this as a *homology plane*.

We investigate homology planes via their compactifications. The compactifications give rise to an algorithmic construction of surfaces from curves in the projective plane $\boldsymbol{P}^2$. If X is a projective surface and $C\subset X$ a curve we call (X, C) or X a *compactification* of any surface V which is isomorphic to the complement $X\backslash C$.

Ramanujam produced the first example of a homology plane. (See RAMANUJAM [1971]). His homology plane was in fact contractible and produced a counterexample to the conjecture that a smooth contractible affine surface over the complex numbers was the standard plane. Any smooth affine variety (over the reals) is the interior of a smooth manifold with boundary. In the case of a homology plane, it follows that the boundary is a homology sphere. When the homology plane is contractible, this observation gives homology planes bounding contractible manifolds—a point of interest in topology. It wasn't until 1987 that other homology planes were found. Gurjar-Miyanishi (see GURJAR—MIYANISHI [1987]) produced all homology planes of logarithmic Kodaira dimension 1. In this paper they ask whether there are an infinite number of contractible homology planes of Kodaira dimension 2. We announced the first affirmative solution to this problem in TOM DIECK—PETRIE [1989]. Among the results of this paper are the details of the announcement. Infinite families of homology planes of Kodaira dimension 2 result from the main theorems A and B here. See (3.20) and (3.21).

Another application of the main results here is the production of homology planes which have non trivial finite order automorphisms. These planes and finite order automorphisms produce counterexamples to the conjecture that