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Osaka J. Math.
30 (1993), 783-811

ASYMPTOTICS OF EIGENVALUES OF THE LAPLACIAN WITH SMALL SPHERICAL ROBIN BOUNDARY

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(Received June 8, 1992)

1. Introduction

Let Ω be a bounded domain in $\boldsymbol{R}^N$ with C^∞ boundary $\partial\Omega$. Let $\tilde{w}$ be a fixed point in Ω and $B(\varepsilon, \tilde{w})$ be the ball of radius ε with the center $\tilde{w}$. We put $\Omega_\varepsilon=\Omega\backslash\overline{B(\varepsilon, \tilde{w})}$. Consider the following eigenvalue problem

$$
(1.1)\qquad \begin{aligned}
-\Delta u(x) &= \lambda u(x) && x\in\Omega_\varepsilon \\
u(x) &= 0 && x\in\partial\Omega \\
u(x)+k\varepsilon^\sigma\frac{\partial u}{\partial\nu_x}(x) &= 0 && x\in\partial B(\varepsilon, \tilde{w})\,.
\end{aligned}
$$

Here k denotes a positive constant. And σ is a real number. Here $\partial/\partial\nu_x$ denotes the derivative along the exterior normal direction with respect to Ω_ε.

Let $\mu_j(\varepsilon)>0$ be the j-th eigenvalue of (1.1). Let μ_j be the j-th eigenvalue of the problem

$$
(1.2)\qquad \begin{aligned}
-\Delta u(x) &= \lambda u(x) && x\in\Omega \\
u(x) &= 0 && x\in\partial\Omega\,.
\end{aligned}
$$

Let $G(x,y)$ (resp, $G_\varepsilon(x, y)$) be the Green function of the Laplacian in Ω (resp. Ω_ε) associated with the boundary condition (1.2) (resp. (1.1)).

Main aim of this paper is to show the following Theorems. Let $\varphi_i(x)$ be the L^2-normalized eigenfunction associated with μ_j. We have the following.

Theorem 1. *Assume $N=3$. We fix j and $\sigma\geqq 1$. Suppose that μ_j is simple. Then, for any fixed $s\in(0, 1)$,*

$$
(1.3)\qquad \begin{aligned}
\mu_j(\varepsilon) &= \mu_j+P_j\varepsilon+O(\varepsilon^{2-s}) && (\quad \sigma\geqq 2) \\
\mu_j(\varepsilon) &= \mu_j+P_j\varepsilon+O(\varepsilon^{\sigma}) && (1<\sigma<2) \\
\mu_j(\varepsilon) &= \mu_j+(1+k)^{-1}P_j\varepsilon+O(\varepsilon^{2-s}) && (\quad \sigma=1)\,,
\end{aligned}
$$

where

$$
P_j = 4\pi\varphi_j(\tilde{w})^2\,.
$$