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# FACTORIZATION OF DOUBLE TRANSFER MAPS

Dedicated to Professor Seiya Sasao on his 60th birthday

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## 1. Introduction

In [6] and [4], the authors have studied a factorization of the double $S^1$-transfer map through the second stage of the chromatic filtration. In this paper, I show that such a factorization exists for other double transfer maps.

Let $\alpha$ be an orientable vector bundle of fiber dimension $a$ over a connected finite complex $X$, and $X^\alpha$ denote the Thom space of $\alpha$. Then we have a cofiber sequence

$$S^a \xrightarrow{i} X^\alpha \xrightarrow{j} X^\alpha/S^a \xrightarrow{\tau} S^{a+1}, \tag{1.1}$$

where $i$ is the inclusion to the bottom sphere. Then, by [7], the $S^1$-transfer map is stably homotopic to $\tau$ when $X=CP^n$ and $\alpha=-\xi$ for the canonical $C$-line bundle $\xi$ over the complex projective space $CP^n$. If $X=\Sigma W$ a suspension of a space $W$, then $\tau$ is stably homotopic to the stable $J$-map $J(\alpha): X\to S^1$. Thus, generalizing the original meaning of transfer maps, we call $\tau$ in (1.1) a transfer map. Then the following stable map $\tau_2$ is called to be a double transfer map.

$$\tau_2 = \tau\wedge\tau: X^\alpha/S^a \wedge Y^\beta/S^b \to S^{a+b+2}, \tag{1.2}$$

where $\beta$ is an orientable vector bundle of fiber dimension $b$ over a connected finite complex $Y$.

By Ravenel [11] a geometric realization of the chromatic filtration has been given, and we shall denote the first two stages in it by

$$\cdots \to \Sigma^{-2}N_2 \xrightarrow{\delta_2} \Sigma^{-1}N_1 \xrightarrow{\delta_1} S^0. \tag{1.3}$$

Here, the spectra are localized at a prime $p$, and there is some difference in our treatment between the cases of an odd prime $p$ and $p=2$. This difference is caused by the use of $K$-theory, and thus we treat the $K$-spectrum $K_\Lambda$ which denotes the complex $K$-spectrum $K_{(p)}$ localized at $p$ in case of an odd prime $p$ and the real $K$-spectrum $KO_{(2)}$ localized at 2 in case of $p=2$. Then we shall show the following:

**Theorem 1.4.** *Let $\tau_2$ be the double transfer map of* (1.2), *and $N_2$ the second*