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ON DELTA-UNKNOTTING OPERATION

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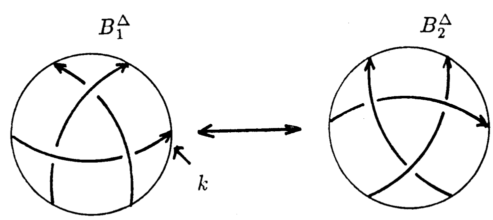
1. Statement of Theorem. In this paper we study oriented knots in the oriented 3-sphere S^3. In [3], H. Murakami and Y. Nakanishi defined a Δ-unknotting operation and proved that any knot can be transformed into a trivial knot by a finite sequence of Δ-unknotting operations. Let k be a knot in S^3 and B_1^{Δ} a 3-ball which intersects k as illustrated in Figure 1(a). Then k_{Δ} denotes the knot in S^3 obtained from k by changing B_1^{Δ} to B_2^{Δ} as illustrated in Figure 1(b). k_{Δ} is said to be obtained from k by a *Δ-unknotting operation.*



(a) (b)

Figure 1

Let Δ_1 and Δ_2 be two Δ-unknotting operations for k such that $k_{\Delta_1} \cong k_{\Delta_2}$. Then Δ_1 and Δ_2 are said to be *homeomorphic,* if there is a homeomorphism h: $S^3 \to S^3$ such that $h(k)=k$, $h(k_{\Delta_1})=k_{\Delta_2}$, $h(B_1^{\Delta_1})=B_1^{\Delta_2}$, and $h(B_2^{\Delta_1})=B_2^{\Delta_2}$.

REMARK. For an ordinary unknotting operation, the following results are known. If the image of an ordinary unknotting operation is unknot, then T. Kobayashi [2], Scharlemann and A. Thompson [4] proved that the number of homeomorphism classes for a non-trivial doubled knot is one. K. Taniyama [5] proved for two-bridge knots, the number is at most two. In constract to such knots, Y. Nakanishi conjectured that for any natural number n, there exist knots such that the number of homeomorphism classes is at least n. A. Kawauchi proved that affirmatively by using imitation theory [1].

Theorem. *Let k be a knot in S^3. Suppose that k_{Δ} is obtained from k by a Δ-unknotting operation. Then the number of the homeomorphism classes of*