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COMPACT SIMPLE LIE ALGEBRAS WITH TWO INVOLUTIONS AND SUBMANIFOLDS OF COMPACT SYMMETRIC SPACES II

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Introduction. This is a continuation of Part I, which appears in the same Journal.

In the previous paper we take a Grassmann bundle $G_s(TM)$ over a compact simply connected irreducible riemannian symmetric space M and consider a G-orbit $\mathcal{V}$ in $G_s(TM)$ by the isometry group G of M. For each $\mathcal{V}$ we can define a class of submanifolds in M, so is called, a *$\mathcal{V}$-geometry*. We moreover assume that $\mathcal{V}$ is a G-orbit which contains an s-dimensional strongly curvature-invariant subspace. Then $\mathcal{V}$ corresponds to a PSLA $(\mathfrak{g}, \sigma, \tau)$ of compact semisimple Lie algebra $\mathfrak{g}$ and two commutative involutions σ, τ. PSLA's are algebraicaly divided into those of inner type and those of outer type.

Our aim in this article is to prove the following

Main Theorem. *Let M be an irreducible compact simply connected riemannian symmetric space and $\mathcal{V}$ a G-orbit of inner type. Then the Lie algebra $\mathfrak{g}$ of Killing vector fields on M is compact simple and the following hold for $\mathfrak{g}$ of classical type:*

(1) *Let $\mathfrak{g}$ be the Lie algebra of type $A_l, l \geq 1$. In this case the $\mathcal{V}$-geometry admits non-totally geodesic $\mathcal{V}$-submanifolds if and only if it is one of the $\mathcal{V}$-geometries in Example* 2, (1).

(2) *Let $\mathfrak{g}$ be the Lie algebra of type $B_l, l \geq 2$. In this case the $\mathcal{V}$-geometry admits non-totally geodesic $\mathcal{V}$-submanifolds if and only if it is one of the $\mathcal{V}$-geometries in Example* 1 (*m : even and r : even*).

(3) *Let $\mathfrak{g}$ be a Lie algebra of type $C_l, l \geq 3$. In this case the $\mathcal{V}$-geometry admits non-totally geodesic $\mathcal{V}$-submanifolds if and only if it is one of the $\mathcal{V}$-geometries in Example* 3, (2).

(4) *Let $\mathfrak{g}$ be the Lie algebra of type $D_l, l \geq 4$. In this case the $\mathcal{V}$-geometry does not admit non-totally geodesic $\mathcal{V}$-submanifolds.*

Examples appeared here are known ones as $\mathcal{V}$-geometries in rank one sym-

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