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COMPACT SIMPLE LIE ALGEBRAS WITH TWO INVOLUTIONS AND SUBMANIFOLDS OF COMPACT SYMMETRIC SPACES I

Dedicated to Professor Masaru Takeuchi on his sixtieth birthday

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Introduction. Let M be a smooth manifold of dimension m and s an integer such that $1 \leq s \leq m-1$. Let $G_s(T_p M)$ be the set of s-dimensional linear subspaces in the tangent space $T_p M$ at p and denote by $G_s(TM)$ the corresponding Grassmann bundle over M, i.e., $G_s(TM) = \cup_{p \in M} G_s(T_p M)$. Harvey-Lawson [4] introduces the notion of *Grassmann geometries*, which is described as follows. Give an arbitary subset $\mathcal{V}$ of $G_s(TM)$. An s-dimensional connected submanifold S of M is called a *$\mathcal{V}$-submanifold* if at each point p of S the tangent space $T_p S$ belongs to $\mathcal{V}$. The collection of $\mathcal{V}$-submanifolds constitutes the *$\mathcal{V}$-geometry*. Grassmann geometries are the collective name of such $\mathcal{V}$-geometries.

We consider $\mathcal{V}$-geometries of the following type. Assume that M is a compact simply connected riemannian symmetric space and denote by G the group of isometries on M. The Lie group G acts transitively on M and at the same time acts on $G_s(TM)$ via the differentials of isometries. As $\mathcal{V}$ we take a G-orbit by this action. The G-orbit $\mathcal{V}$ is a homogeneous bundle over M with homogeneous fibres, i.e., $\mathcal{V} = \cup_{p \in M} \mathcal{V}_p$, $\mathcal{V}_p = \mathcal{V} \cap G_s(T_p M)$, and G acts transitively on the family of fibres $\mathcal{V}_p$. Moreover the isotropy subgroup K_p in G at p acts transitively on the fibre $\mathcal{V}_p$. Roughly speaking, Grassmann geometries of this type correspond to classes of submanifolds with congruent tangent space. From this point of view the Grassmann geometries are important for us to study the submanifold theory of riemannian symmetric space.

In this article we especially treat the following G-orbits. Denote by R the curvature tensor on M. An s-dimensional linear subspace V in $T_p M$ is called *strongly curvature-invariant* if it satisfies that

$$(0.1) \qquad R_p(V, V)\, V \subset V \quad \text{and} \quad R_p(V^{\perp}, V^{\perp})\, V^{\perp} \subset V^{\perp},$$

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