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MEASURED HAUSDORFF CONVERGENCE OF RIEMANNIAN MANIFOLDS AND LAPLACE OPERATORS

dedicated to Prof. Hideki Ozeki on his 60th birthday

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0. Introduction

K. Fukaya introduced in [11] a topology on a set of metric spaces equipped with Borel measures, called the measured Hausdorff topology and discussed the continuity of the eigenvalues of the Laplace operators of Riemannian manifolds with uniformly bounded curvature. The purpose of the present paper is to study more closely the Laplace operators of Riemannian manifolds which collapse in this topology to a space of lower dimension while keeping their curvature bounded.

0.1. According to [16], a map $h: X \rightarrow Y$ of metric spaces is said to be an ε-Hausdorff approximation if $|\mathrm{dis}(x, x')-\mathrm{dis}(h(x), h(x'))| \leqq \varepsilon$ for all $x, x' \in X$ and the ε-neighborhood of the image $h(X)$ coincides with Y. A sequence of compact metric spaces $\{X_i\}$ converges, by definition, to a compact metric space Y in the Hausdorff distance if there are a sequence of positive numbers $\{\varepsilon(i)\}$ going to zero as i tends to infinity and $\varepsilon(i)$-Hausdorff approximations $h_i: X_i \rightarrow Y$ of X_i into Y. Moreovre when each metric space X_i is equipped with a Borel measure μ_i of unit mass, according to [11], we say that $\{(X_i, \mu_i)\}$ converges to Y with a Borel measure μ_∞ of unit mass in the measured Hausdorff topology, if in addition, these maps $h_i: X_i \rightarrow Y$ are Borel measurable and the push-forward measure $h_{i*}\mu_i$ converges to μ_∞ in the weak* topology.

Now we shall consider a sequence of compact Riemannian manifolds $\{(M_i, g_i)\}$ of dimension m whose sectional curvature K_{M_i} is bounded uniformly in its absolute value by a constant, say 1, and assume that tihs sequence converges to a compact metric space M_∞ in the Hausdorff distance. When the volume of M_i is bounded uniformly away from zero by a positive constant, Gromov's

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