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J-GROUPS OF THE SUSPENSIONS OF THE STUNTED LENS SPACES MOD 2p

Dedicated to Professor Michikazu Fujii on his 60th birthday

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1. Introduction

Let $L^n(q)=S^{2n+1}/\boldsymbol{Z}_q$ be the (2n+1)-dimensional standard lens space mod q. As defined in [10], we set

$$(1.1)\qquad \begin{aligned} L_q^{2n+1} &= L^n(q)\,, \\ L_q^{2n} &= \{[z_0, \cdots, z_n]\in L^n(q)\,|\, z_n \text{ is real} \geq 0\}\ . \end{aligned}$$

By the several papers, we determined the KO-groups $\widetilde{KO}(S^j(L_q^m/L_q^n))$ of the suspensions of the stunted lens spaces L_q^m/L_q^n for the cases $j\equiv 1 \pmod 2$ [25], $q=2$ [12], $q=4$ [20] and $q=8$ [21]. Moreover we determined the J-groups $\tilde{J}(S^j(L_q^m/L_q^n))$ for the cases odd primes q [19], $q=2$ [18], $q=4$ [20] and $q=8$ [21]. The purpose of this paper is to determine the KO-groups $\widetilde{KO}(S^j(L_{2p}^m/L_{2p}^n))$ and J-groups $\tilde{J}(S^j(L_{2p}^m/L_{2p}^n))$ for odd primes p.

This paper is organized as follows. In section 2 we state the main theorems: Theorem 2 gives a direct sum decomposition of $\widetilde{KO}(S^j(L_{2^rq}^m/L_{2^rq}^n)$ for $j\equiv 0$ (mod 2), Theorem 3 gives a direct sum decomposition of $\tilde{J}(S^j(L_{2^rq}^m/L_{2^rq}^n))$ for $j\equiv 0$ (mod 2) and $n+j+1\not\equiv 0$ (mod 4), Theorem 4 gives the structure of $\tilde{J}(S^j(L_{2p}^m/L_{2p}^n))$ for $j\equiv 0$ (mod 2) and $n+j+1\equiv 0$ (mod 4) and the necessary conditions for L_{2p}^m/L_{2p}^n and $L_{2p}^{m+t}/L_{2p}^{n+t}$ to be of the same stable homotopy type are given by Theorem 5 which is an application of Theorems 3 and 4. In section 3 we prepare some lemmas and recall known results in [12], [19] and [25]. The proofs of Theorem 2 and Theorem 3 are given in section 4. The proof of Theorem 4 is given in section 5. In the final section we give the proof of Theorem 5.

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2. Statement of results

Let $\nu_p(s)$ denote the exponent of the prime p in the prime power decomposition of s, and $\mathfrak{m}(s)$ the function defined on positive integers as follows (cf.