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A DECOMPOSITION OF $BP\langle 2\rangle$ AND v_1-TORSION

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Introduction

For any p-local connective spectrum F (with p a prime number), the first author discovered in [2] integers ρ_j and maps $F\to\Sigma^{j+1}H(\pi_jF, 0)$ between F and Eilenberg-MacLane spectra such that the compositions

$$F \longrightarrow \Sigma^{j+1}H(\pi_jF, 0) \xrightarrow{p^{\rho_j}} \Sigma^{j+1}H(\pi_jF, 0)$$

are trivial. This enabled him to prove that in the Atiyah-Hirzebruch-Dold spectral sequence for the F-homology of any bounded below spectrum, $p^{\rho_j}d^{j+1}_{s,t}=0$ for all $j\geq 1$, s and t. Now, let us consider the Brown-Peterson spectrum BP with $BP_*=\mathbf{Z}_{(p)}[v_1, v_2, \cdots]$, where the degree of v_k is $|v_k|=2(p^k-1)$ for $k\geq 1$, and denote as usual by $BP\langle m\rangle$ the spectrum such that $BP\langle m\rangle_*\cong\mathbf{Z}_{(p)}[v_1\ v_2, \cdots, v_m]$ for any $m\geq 1$. This paper exploits a similar composite

$$BP\langle 2\rangle/(v_2^j) \longrightarrow \Sigma^{j|v_2|+1}BP\langle 1\rangle \xrightarrow{v_1^{(p+1)j+1}} \Sigma^{-|v_1|+1}BP\langle 1\rangle$$

which, as a consequence of calculations by the second author in [6] can be seen to be trivial. As a result, we can construct maps

$$f_j\colon BP\langle 2\rangle \to \Sigma^{-|v_1|}BP\langle 1\rangle\,,$$

for all $j\geq 1$, which we control on the homotopy level (see Theorem 2.1). These maps induce maps between the Atiyah-Hirzebruch-Dold spectral sequences for $BP\langle 2\rangle$ and $BP\langle 1\rangle$-homology respectively which provide information about the differentials in the Atiyah-Hirzebruch-Dold spectral sequence for $BP\langle 2\rangle$ (see Theorem 3.3). On the other hand, the triviality of the above composition implies torsion results on the differentials in a modified Bockstein spectral sequence for $BP\langle 2\rangle$ analogous to the BP Bockstein spectral sequence of Johnson and Wilson [5] (see Theorem 4.5).

In order to illustrate how this new information might be used in calculation,

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