LEFT MONOMIAL RINGS-A GENERALIZATION OF MONOMIAL ALGEBRAS

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0. Introduction

A particularly tractable class of finite dimensional algebras defined by quivers and relations is that of monomial algebras, i.e., those for which the ideal of relations is generated by a collection of paths. The homological structure of these algebras is very well understood and some constructions for them are even algorithmic. There is, for example, an algorithm due to Green, Happel and Zacharia (see [11], where the algebras are called 0-relations algebras) for constructing the projective resloutions of the simple modules which determines their projective dimensions in a predictable number of steps. The Cartan determinant conjecture is known to be true for these algebras since they are positively graded ([18]) and the finitistic dimensions are finite ([12] and [13]) and are thoroughly understood due to the recent work of B.Z. Huisgen ([13] and [14]). Other properties of monomial algebras will be cited below.

Here we introduce a class of left artinian rings which includes that of monomial algebras and we show that many of the above results remain valid within it. The proposed rings, called *left monomial rings* (see Definition 2.2) will include monomial algebras and the more general 0-relations algebras given by species and 0-relations, as well as left (almost) serial rings, right serial rings, hereditary artinian rings and more. To each such ring R is associated a monomial algebra A so that, in many ways, R and A have the "same" homological properties (see Theorem 2.3); enough so that, for example, the projective dimensions of the corresponding simple modules are the same. (See Theorem 2.3 and its corollary.)

1. Tree modules

We fix throughout a basic left artinian ring R with radical J. In the sequel

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