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FLAT TORSIONFREE MODULES AND QF-3 RINGS

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Since Thrall [25] proposed the concept of QF-3 algebra as a generalization of QF algebras, several extensions of this concept have been proposed for general rings. Perhaps the most spread notion of QF-3 ring is the following: A ring R is called a left QF-3 ring if it has a minimal faithful left R-module (see [23]). However, other authors proposed alternative notions, mainly for noetherian rings. We will look at two of them that seem very intersting. The first one is due to Morita [16] and it has been investigated recently by Hoshino [9], [10]. A ring R is said to be left Morita-QF-3 ring (shortly, MQF-3) if $E(_RR)$ is a flat left R-module. The second one was proposed by Sumioka [20], [21], [22]. The ring R is called a left Sumioka-QF-3 ring (shortly, left SQF-3) if every finitely generated submodule of $E(_RR)$ is torsionless. Every commutative domain is MQF-3 and SQF-3. It is well-known that in the case of left Artinian rings, these three concepts are equivalent and, moreover, they are right-left symmetric.

The aim of Section 2 is to find relations between the different extensions of QF-3 rings mentioned above. As a consequence of the main result of Section 2 (Theorem 2.7) we will show that if R has D.C.C. on rationally closed left ideals then R is left or right MQF-3 if and only if R is left or right SQF-3 (Corollary 2.8). Moreover, these rings are precisely those that Masaike characterized [14, Theorem 2] as the rings with a semi-primary QF-3 two sided maximal quotient ring.

The unifying idea to prove these results comes from the problem of existence of flat covers [4], [5]. In connection with this problem we proposed in [7] to investigate the rings R for which the class $\mathcal{F}_0$ of the submodules of flat left R-modules is a torsionfree class. To be exact, we say that R is a left FTF ring if there is a hereditary torsion theory τ_0 on R-Mod such that $\mathcal{F}_0$ is the class of all τ_0-torsionfree left R-modules. The key result is that the left MQF-3 (or left SQF-3) rings with D.C.C. on rationally closed left ideals are precisely the τ_0-artinian left FTF rings.

A ring R is said to be a left IF ring if every injective left R-module is flat. In other words, $\mathcal{F}_0=R$-Mod. Thus, IF rings are the "trivial" FTF rings.

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