Morinaka, H.
Osaka J. Math.
30 (1993), 331–348

ON THE REPRESENTATION OF POTENTIALS BY A GREEN FUNCTION AND THE PROPORTIONALITY AXIOM ON P-HARMONIC SPACES

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(Received January 23, 1992)
(Revised September 2, 1992)

Introduction

Let $(X, \mathcal{U})$ be a P-harmonic space in the sense of Constantinescu-Cornea [5] with a countable base. If we assume the Doob convegence property, the proportionality axiom and the condition (A) (see [7]) on $(X, \mathcal{U})$, then $(X, \mathcal{U})$ has a Green function and every potential on X is represented by the Green function with a Radon measure on X [7]. In this paper we consider the following problem: if we assume the existence of a Green function $k(x, y)$ on $(X, \mathcal{U})$, what conditions derive the representation of all potentials by the function $k(x, y)$ with a Radon measure on X.

In 1979, A. Boukricha [3] proved that if any finite continuous potential with compact harmonic support is represented by the function $k(x, y)$ then the proportionality axiom is satisfied. Furthermore, U. Schirmeier [9] proved that if at least one bounded continuous strict potential is represented by $k(x, y)$ then the proportionality axiom is satisfied. In 1982, for a Brelot space H. Ben Saad [1] proved that if the function $k(x, y)$ is symmetric i.e. $k(x, y)=k(y, x)$ or if there exists an adapted cone P' of continuous potentials which are represented by $k(x, y)$ such that $P'-P'$ is uniformly dense in $C(K)$ for an arbitrary compact subset $K\subset X$ then every potential on X can be represented by the function $k(x, y)$ and also the proportionality axiom is satisfied.

We shall show, first of all in section 2 that for a general P-harmonic space with a countable base which admits a Green function $k(x, y)$, if the convex cone P_1 of all continuous potentials which are represented by $k(x, y)$ is adapted, inf-stable and separates points of X, we obtain the same conclusions as those of H. Ben Saad. In section 3, we shall show that if there exists a second P-harmonic space $(X, \mathcal{U}^*)$ which has the Green function $k^*(x, y)=k(y, x)$ then the convex cone P_1 possesses the above properties. In section 4, we show that the assumptions of A. Boukricha and U. Schirmeier for concluding the proportionality axiom coincide with the assumption that the convex cone P_1 possesses