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CHARACTERIZATION OF CONDITIONAL EXPECTATIONS FOR M-SPACE-VALUED FUNCTIONS

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Introduction Let $(\Omega, \mathcal{A}, \mu)$ be a probability space, E a Banach space. We consider constant-preserving contractive projections of $L_1(\Omega, \mathcal{A}, \mu, E)$ into itself. If $E=R$ or E is a strictly-convex Banach space, then it is known (Ando [2], Douglas [3] and Landers and Rogge [6]) that such operators coincide precisely with the conditional expectation operators. If $E=L_1(X, S, \lambda, R)$, where (X, S, λ) is a localizable measure space, then the author [8] proved that such operators which are translation invariant coincide with the conditional expectation operators. If $E=L_\infty(X, S, \lambda, R)$, where (X, S, λ) is a measure space, and the dimension of E is bigger than 2, then author [9] proved that such operators coincide with the conditional expectation operators. On the other hand if $E=L_\infty(X, S, \lambda, R)$ and the dimension of E is 2, then the author [9] proved that such operators can be expressed as a linear combination of two conditional expectation operators. In this paper we deal with the case that E is an M-space. An L_∞-space is an M-space, and hence this paper contains the result of the author [9] as a special case. If E is an M-space, whose dimension is bigger than 2, then such operators coincide with conditional expectation operators.
If E is an M-space with unit, i.e., the unit ball in E has a least upper bound, then we can prove many of lemmas in this paper by easier way. In this paper we do not assume that E is an M-space with unit.

1. Definitions and properties of M-spaces.

Let E be a real linear space and N the class of natural numbers and R the class of real numbers.

DEFINITION 1.1. A lattice $(E, \leqq)$ is an ordered linear space such that
(1) $a \leqq a$ for any $a \in E$;
(2) if $a, b \in E$, $a \leqq b$ and $b \leqq a$, then $a=b$;
(3) if $a, b, c \in E$ and $a \leqq b$ and $b \leqq c$, then $a \leqq c$;
(4) if $a \leqq b$, then $a+c \leqq b+c$ for any $c \in E$;
(5) if $0 \leqq a$ in E, then $0 \leqq ka$ in E for any $k \geqq 0$ in R;
(6) $\sup\{a, b\}$ and $\inf\{a, b\}$ exist for any $a, b \in E$.