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# THE EIGENVALUE DISTRIBUTION OF ELLIPTIC OPERATORS WITH HÖLDER CONTINUOUS COEFFICIENTS II

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## 1. Introduction

This is the continuation of the previous paper [11], in which we attempted the improvement of the remainder estimate for the eigenvalue distribution of the elliptic operator of order $2m$ with Hölder continuous coefficients of top order. We use the same notation as in [11] if not specified.

Let us recall the situation. Let $\Omega$ be a bounded domain in $\boldsymbol{R}^n$. We consider a symmetric integro-differential sesquilinear form

$$B[u, v]=\int_{\Omega} \sum_{|\alpha|,|\beta| \leqq m} a_{\alpha\beta}(x) D^{\alpha} u(x) \overline{D^{\beta} v(x)}\, dx$$

and a closed subspace $V$ of the Sobolev space $H^m(\Omega)$, and assume the following.

(H1) $H_0^m(\Omega) \subset V \subset H^m(\Omega)$.

(H2) There exist $C_0 \geqq 0$ and $\delta_0>0$ such that

$$B[u, u] \geqq \delta_0\|u\|_m^2-C_0\|u\|_0^2 \qquad \text{for any} \quad u \in V.$$

(H3) The coefficients $a_{\alpha\beta}(x)(|\alpha|+|\beta| \leqq 2m)$ are bounded on $\Omega$, and for some $\tau>0$ the coefficients of top order satisfy

$$a_{\alpha\beta} \in \mathscr{B}^{\tau}(\Omega) \quad (|\alpha|=|\beta|=m).$$

REMARK 1.1. Since an element of $\mathscr{B}^{\tau}(\Omega)$ can be extended to an element of $\mathscr{B}^{\tau}(\boldsymbol{R}^n)$ (see [10], [20]), we may assume that

$$(1.1) \qquad \sum_{|\alpha|=|\beta|=m} a_{\alpha\beta}(x) \xi^{\alpha+\beta} \geqq \delta_0|\xi|^{2m} \qquad \text{for} \quad x \in \boldsymbol{R}^n, \xi \in \boldsymbol{R}^n$$

by modifying the values of $a_{\alpha\beta}$ outside $\Omega$ and replacing $\delta_0$ with another constant if necessary.

Let $\mathcal{A}$ be the self-adjoint operator associated with the variational triple $\{B, V, L_2(\Omega)\}$, and let $N(t)$ $(=N(t, B, V, L_2(\Omega)))$ denote the number of the eigen-