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NONCOPRIME ACTION AND CHARACTER CORRESPONDENCES

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1. Introduction

In [7], Nagao extended the Glauberman Correspondence to the non-coprime case by restricting the attention to the S-invariant p-defect zero characters of a finite group G acted by a finite p-group S. Concretely, if G is a complemented normal subgroup of Γ and C is a set of representatives of G-conjugacy classes of complements of G in Γ, Nagao showed that there exists a natural bijection from the set of Γ-invariant p-defect zero characters of G onto $\cup_{S\in C}\{p$-defect zero characters of $C_G(S)\}$, whenever Γ/G is a p-group.

Now we want to make no assumptions on Γ/G (although we will end up making some assumptions on G) and still show that there exists a natural map from some subset of the Γ-invariant characters of G (those who have p-defect zero for the primes dividing $|\Gamma/G|$) into $\cup_{S\in C}\mathrm{Irr}(C_G(S))$.

As we mention, we pay for this extra generality: we impose some conditions on G (G must be π-separable for the set of primes π dividing $|\Gamma/G|$). Also, although defect zero characters of G will map into defect zero characters of $C_G(S)$ it will not be true, in general, that our map is onto (think on a π-group acted by another π-group with trivial fixed points subgroup). This will be the case, however, when the Hall π-subgroups of Γ are nilpotent (as it happens in Nagao's case). When Γ/G is a p-group (and G is p-solvable) we will certainly show that our map coincides with Nagao's.

The key point in this note is to consider an interesting subset of the irreducible characters of a finite group G acted by a finite group S whose order is nonnecessarily coprime to $|G|$. If $\mathrm{Ind}_S(G)=\{\chi\in\mathrm{Irr}(G)$ such that $\chi=\mu^G$, where μ is an S-invariant character of an S-invariant subgroup H of G with order coprime to $S\}$, then there exists a natural one to one map from $\mathrm{Ind}_S(G)$ into $\mathrm{Irr}(C_G(S))$. We will show that the image of $\chi\in\mathrm{Ind}_S(G)$ is $\mu^{*C_G(S)}$, where $\mu^*\in\mathrm{Irr}(C_H(S))$ is the Glauberman-Isaacs correspondent of $\mu\in\mathrm{Irr}_S(H)$. Of course, one of the problems in this note will be to show that if μ induces irreducibly to G, then μ^* induces irreducibly to $C_G(S)$ (this was done in [6] when the

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