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ON AUSLANDER-REITEN COMPONENTS FOR CERTAIN GROUP MODULES

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Let G be a finite group and k a field of characteristic $p>0$. Let $\Gamma_s(kG)$ be the stable Auslander-Reiten quiver of the group algebra kG. By Webb's theorem , the tree class of a connected component Δ of $\Gamma_s(kG)$ is a Euclidean diagram, a Dynkin diagram or one of the infinite trees A_∞, B_∞, C_∞, D_∞, or A_∞^∞. Moreover if Δ contains the trivial kG-module k, then the graph structure of Δ has been investigated (see [21], [16] and [17]). In this paper we study a connected component of $\Gamma_s(kG)$ containing an indecomposable kG-module whose k-dimension is not divisible by p. Suppose that M is an indecomposable kG-module and $p \not| \dim_k M$. In Section 2, we will show that M lies in a connected component isomorphic to $\boldsymbol{Z}A_\infty$ if k is algebraically closed and a Sylow p-subgroup of G is not cyclic, dihedral, semidihedral or generalized quaternion. In Section 3 we make some remarks on tensoring the component containing the trivial kG-module k with M. In Sections 4 and 5 we consider the situation where $p=2$ and a Sylow 2-gubgroup of G is dihedral of order at least 8 or semidihedral.

The notation is almost standard. All modules considered here are finite dimensional over k. We write $W\cong W'$ (*mod projectives*) for kG-modules W and W' if the projective-free part of W is isomorphic to that of W'. For an indecomposable non-projective kG-module W, we write $\mathcal{A}(W)$ to denote the Auslander-Reiten sequence (AR-sequence) $0\to\Omega^2W\to m(W)\to W\to 0$ terminating at W, where Ω is the Heller operator, and we write $m(W)$ to denote the middle term of $\mathcal{A}(W)$. If an exact sequence of kG-modules $\mathcal{S}$ is of the form $0\to\Omega^2W\oplus U'\to m(W)\oplus U\oplus U'\to W\oplus U\to 0$, where W is an indecomposable non-projective kG-module, and U,U' are proejctive or 0, we say that $\mathcal{S}$ is the AR-sequecne $\mathcal{A}(W)$ *modulo projectives*. The symbol $\otimes$ denotes the tensor product over the coefficient field k. For an exact sequence of kG-modules $\mathcal{S}$: $0\to A\to B\to C\to 0$ and a kG-module W, we write $\mathcal{S}\otimes W$ to denote the tensor sequence $0\to A\otimes W\to B\otimes W\to C\otimes W\to 0$. Concerning some basic facts and terminologies used here, we refer to [2], [10] and [11].