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HIGH DEGREE ANTI-INTEGRAL EXTENSIONS OF NOETHERIAN DOMAINS

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Introduction. Let R be a Noetherian integral domain and $R[X]$ a polynomial ring. Let α be an element of an algebraic field extension L of the quotient field K of R and let $\pi: R[X]\to R[\alpha]$ be the R-algebra homomorphism sending X to α. Let $\varphi_\alpha(X)$ be the monic minimal polynomial of α over K with $\deg \varphi_\alpha(X)=d$ and write $\varphi_\alpha(X)=X^d+\eta_1X^{d-1}+\cdots+\eta_d$. Let $I_{[\alpha]}:=\cap_{i=1}^d(R:_R \eta_i)$. For $f(X)\in R[X]$, let $C(f(X))$ denote the ideal generated by the coefficients of $f(X)$. Let $J_{[\alpha]}: =I_{[\alpha]}\, C(\varphi_\alpha(X))$, which is an ideal of R and contains $I_{[\alpha]}$. The element α is called an anti-integral element of degree d over R if $\mathrm{Ker}\,\pi= I_{[\alpha]}\,\varphi_\alpha(X)\,R[X]$. When α is an anti-integral element over R, $R[\alpha]$ is called an anti-integral extension of R. In the case $K(\alpha)=K$, an anti-integral elemet α is the same as an anti-integral element (i.e., $R=R[\alpha]\cap R[1/\alpha]$) defied in [5]. The element α is called a super-primitive element of degree d over R if $J_{[\alpha]}\not\subset p$ for all primes p of depth one.

For $p\in\mathrm{Spec}(R)$, $k(p)$ denotes the residue field R_p/pR_p and $\mathrm{rank}_{k(p)}\,R[\alpha]\otimes_R k(p)$ denotes the dimension as a vector space over $k(p)$. We are interested in characterizing the flatness and the integrality of an anti-integral extension $R[\alpha]$ of R. Indeed, among others we obtain the following results:

(i) $R[\alpha]$ is flat over R if and only if $\mathrm{rank}_{k(p)}\,R[\alpha]\otimes_R k(p)\leq d$ for all $p\in\mathrm{Spec}(R)$,

(ii) $R[\alpha]$ is integral over R if and only if $\mathrm{rank}_{k(p)}\,R[\alpha]\otimes_R k(p)=d$ for all $p\in\mathrm{Spec}(R)$.

Thus if an anti-integral extension $R[\alpha]$ is integral over R, then $R[\alpha]$ is flat over R. Concerning a super-primitive element, we obtain that if R is a Krull domain and α is an algebraic element over R, then α is a super-primitive element. We also obtain that a super-primitive element is an anti-integral element. More precisely, α is super-primitive over R if and only if α is anti-integral over R and $R[\alpha]_p$ is flat over R_p for any prime ideal p of depth one.

Using these results, we obtain the following:

Let $\Delta(S)$ denote the set $\{p\in\mathrm{Spec}(R)\,|\,\mathrm{rank}_{k(p)}S\otimes_R k(p)=d\}$, where S is an extension of R of degree d and let $Dp_1(R)$ denote the set of all prime ideals of R of depth one. Assume that $[L: K]=d$, and that $\alpha_1, \cdots, \alpha_n\in L$ are anti-integral elements of degree d, and let $A=R[\alpha_1, \cdots, \alpha_n]$. If $\Delta(R[\alpha_i])\supset Dp_1(R)$ $(1\leq i\leq n)$