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INVARIANT DIFFERENTIAL OPERATORS ON THE GRASSMANN MANIFOLD $G_{2,n-1}(C)$

TAKESHI SUMITOMO AND KWOICHI TANDAI

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0. Introduction. The present paper is the latter one of twin papers on invariant linear differential operators of Grassmann manifolds. In the former one [9] we determined and clarified the structure of the algebra $\boldsymbol{D}(\boldsymbol{SG}_{2,n-1}(\boldsymbol{R}))$ of invariant linear differential operators on the Grassmann manifold $\boldsymbol{SG}_{2,n-1}(\boldsymbol{R})$ of oriented 2-planes in $\boldsymbol{R}^{n+1}$ by exhibiting a set of generators with their simultaneous eigenspace decompositions.

The complex Grassmann manifold $\boldsymbol{G}_{2,n-1}(\boldsymbol{C})$ defined as the totality of complex 2-planes passing through the origin of $\boldsymbol{C}^{n+1}$, is known to be a symmetric space of rank 2. Hence, the algebra $\boldsymbol{D}(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$ of invariant linear differential operators acting on $C^\infty(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}), \boldsymbol{R})$ is generated by two differential operators, where $C^\infty(M, \boldsymbol{K})$ denotes the algebra of $\boldsymbol{K}$-valued C^∞-functions defined on a complex manifold M and $\boldsymbol{K}$ denotes either the real number field $\boldsymbol{R}$ or the complex number field $\boldsymbol{C}$.

The aim of the present paper lies, as in [9], in exhibiting a simultaneous eigenspace decomposition of an explicit set of generators $\Delta_0^\wedge$ and $\Delta_1^\wedge$ of the algebra $\boldsymbol{D}(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$.

Define

$$\begin{aligned}
\boldsymbol{S}^p(\boldsymbol{P}_n(\boldsymbol{C})) &:= \sum_{k+l=p} \boldsymbol{S}^{k,l}(\boldsymbol{P}_n(\boldsymbol{C})) \quad \text{(direct sum)}, \\
\boldsymbol{S}^*(\boldsymbol{P}_n(\boldsymbol{C})) &:= \sum_{p\geq 0} \boldsymbol{S}^p(\boldsymbol{P}_n(\boldsymbol{C})) \quad \text{(direct sum)}, \\
\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C})) &:= \sum_{k,l\geq 0} \boldsymbol{S}^{k,l}(\boldsymbol{P}_n(\boldsymbol{C})) \quad \text{(direct sum)},
\end{aligned}$$

where $\boldsymbol{S}^{k,l}(\boldsymbol{P}_n(\boldsymbol{C}))$ is the $C^\infty(\boldsymbol{P}_n(\boldsymbol{C}), \boldsymbol{C})$-module of complex (contravariant) symmetric tensor fields of bidegree (k, l) on the complex projective space $\boldsymbol{P}_n(\boldsymbol{C})$. $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$ is a bigraded algebra over $C^\infty(\boldsymbol{P}_n(\boldsymbol{C}), \boldsymbol{C})$. We obtained in [8] the following about the complex projecitve space $(\boldsymbol{P}_n(\boldsymbol{C}), g_0)$ with prescribed standard Riemannian mteric g_0:

(1) The eigenspace decomposition of Δ_0 restricted to $\boldsymbol{K}^{**}(\boldsymbol{P}_n(\boldsymbol{C}), g_0)$ is given, Where Δ_0 is the Lichnerowicz operator acting on $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$ and $\boldsymbol{K}^{**}(\boldsymbol{P}_n(\boldsymbol{C}), g_0)$ is the bigraded $\boldsymbol{C}$-subalgebra of $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$ defined as