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ON HANDLE NUMBER OF SEIFERT SURFACES IN S^3

Dedicated to Professor Hideki Ozeki on his sixtieth birthday

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1. Introduction

Let L be an oriented link in S^3. A *Seifert surface* R for L is a compact oriented surface, without closed components, such that $\partial R=L$. Suppose that the complementary sutured manifold (M, γ) for R (for the definition, see section 4) is irreducible. The *handle number* of R is as follows:

$$h(R) = \min\{h(W); (W, W') \text{ is a Heegaard splitting of } (M; R_+(\gamma), R_-(\gamma))\}.$$

We give the definitions of $h(W)$ and a Heegaard splitting of $(M; R_+(\gamma), R_-(\gamma))$ in section 2 using compression bodies which is introduced by Casson and Gordon in [1].

Note that R is a fiber surface if and only if $h(R)=0$.

In this paper, we completely determine the handle numbers of incompressible Seifert surfaces for prime knots of ≤ 10 crossings. In addition, we show that there is a knot which admits two minimal genus Seifert surfaces whose handle numbers are mutually different (see Example 6.2).

Let R be a Seifert surface in S^3 obtained by a $2n$-Murasugi sum (for the definition, see section 4) of two Seifert surfaces R_1 and R_2 whose complementary sutured manifolds are irreducible. In [7], we have shown the following two theorems:

Theorem A ([7], Theorem 1).

$$h(R_1)+h(R_2)-(n-1)\leq h(R)\leq h(R_1)+h(R_2).$$

Theorem B ([7], Theorem 2). *If R_1 is a fiber surface, then $h(R)=h(R_2)$.*

And it has been also shown in [7] that the estimation in Theorem A is the best possible.

In this paper, we give a sufficient condition to realize the upper equality $h(R)=h(R_1)+h(R_2)$ of Theorem A in the case of a plumbing (i.e., $n=2$). In fact, we prove: