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THE EXACTNESS OF GENERALIZED SKEW PRODUCTS

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0. Introduction

Recently, it appears several papers concerning ergodic properties of random maps i.e. skew products. See T. Morita [5], S. Pelikan [7], etc. The main proof tool in their considerations is the so-called Perron-Frobenius operator. In the present paper the autor proves the theorem about exactness of generalized skew products using the Pinkser algebra.

Let $\sigma: X \to X$ be the shift endomorphism in a space $X \subset \{1, \cdots, s\}^N$ preserving μ. Let $(\tilde{\sigma}, \tilde{\mu})$ be the natural extension of (σ, μ) to the automorphism. The automorphism $\tilde{\sigma}$ is the shift automorphism on the set $\tilde{X} \subset \{1, \cdots, s\}^Z$. Let $\tilde{A}_i = \{\tilde{x} \in \tilde{X}: \tilde{x}(0) = i\}$, $\tilde{\alpha} = \{\tilde{A}_1, \cdots, \tilde{A}_s\}$ and $\tilde{\alpha}_m^n = \bigvee_{k=m}^{n} \tilde{\sigma}^k \tilde{\alpha}$.

DEFINITION. The endomorphism σ is called discrete if $\tilde{\mu}(\tilde{C}) > 0$, for some atom $\tilde{C}$ in $\tilde{\alpha}_{-\infty}^0 \wedge \tilde{\alpha}_0^\infty$.

EXAMPLES: If σ is one-sided Markov shift then it is discrete. If σ is given by Lasota-Yorke type map, then it is also discrete (see [8]).

Let p be a Borel measure on $[0, 1]$ which is positive on open sets. Moreover, let $T_1, \cdots, T_s$ be piecewise monotonic and continuous transformations of $[0, 1]$ into itself so that there exists the partition $\beta_0 = \{I_1, I_2, \cdots\}$ of finite entropy given by $I_i = (t_{i-1}, t_i)$ with $0 = t_0 < t_1 < \cdots$, $\lim t_i = 1$, such that $T_j | (t_i, t_{i+1})$ is continuous and strictly monotonic, for $j = 1, \cdots, s$, $i = 0, 1, \cdots$. We assume that the transformation

$$\bar{T}(x, y) = (\sigma(x), T_{x(0)} y) \tag{1}$$

preserves the product measure $\mu \times p$. Such a transformation is called generalized skew product [2]. The following theorem provides sufficient conditions for $\bar{T}$ to be an exact transformation.

Theorem 1. *Let σ be a discrete endomorphism. If the transformations T_i are 1–1 p a.e., for $i = 1, \cdots, s$, and T_i does not preserve the measure p for some i, then the transformation $\bar{T}$ is exact or $\bar{T}^m$ is not ergodic, for some $m \neq 0$.*

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