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SPECTRA OF RANDOM MEDIA WITH MANY RANDOMLY DISTRIBUTED OBSTACLES

Dedicated to Professor Shigetoshi Kuroda on his 60th birthday

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1. Let Ω be a bounded domain in R^d with smooth boundary $\partial\Omega$. Let $B(\mathcal{E}, w_i)$ $(i=1, \cdots, n)$ be balls of radius $\mathcal{E}$ with centers $w_1, \cdots, w_n$. We consider the eigenvalue problem of the Laplacian in

$$\Omega_{w(m)} = \Omega\backslash\overline{\cup_{i=1}^{n} B(\mathcal{E}, w_i)}$$

under the Dirichlet condition on its boundary. Under some scaling limit $\mathcal{E}\to 0$, $n\to\infty$, $n^{\sigma}\mathcal{E}\to\alpha$ we know that the spectra of $-\Delta$ in $\Omega_{w(m)}$ under the Dirichlet condition on $\partial\Omega_{w(m)}$ tends to the spectra of Schrödinger operator $-\Delta+cV$ in Ω under the Dirichlet condition on $\partial\Omega$.

There are two main directions in previous research works concerning related problems. One is homogenization as was studied in [3], [7], and another direction is to calculate the eigenvalue of $-\Delta$ in $\Omega_{w(m)}$ in statistical setting, the later of which this paper concerns.

Let $V(x)$ be a positive continuous function on $\bar{\Omega}$ satisfying

$$\int_{\Omega} V(x)\,dx = 1\,.$$

Then, Ω can be thought as probability space by the probability law

$$P(x\in A) = \int_{A} V(x)\,dx\,.$$

Let Ω^n be the product probability space; the corresponding probability law is denoted also by P for any n. Fix $\beta\in[d-2, d)$. Setting $\mathcal{E}=m^{-1}$, we take m in place of $\mathcal{E}$ as a parameter. Fix and define $n=[m^{\beta}]$, $\mu_j(w(m))=$the j-th eigenvalue of $-\Delta$ in $\Omega_{w(m)}$ under the Dirichlet condition on $\partial\Omega_{w(m)}$. Each $\mu_j(w(m))$ is viewed as a random variable on Ω^n.

Problem A. Can one say anything about the statistics of $\mu_i(w(m))$ on Ω^n when $m\to\infty$?

We know the following partial answer.