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ON THE LARGE TIME BEHAVIOR OF SOLUTIONS FOR SOME DEGENERATE QUASILINEAR PARABOLIC SYSTEMS

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1. Introduction

We consider the large time behavior of the solutions for the following Cauchy problem:

$$\begin{aligned} u_t &= (u^m)_{xx} - v^n u^n \quad \text{in } \boldsymbol{R}\times(0,\infty) \\ v_t &= (v^m)_{xx} - u^n v^n \quad \text{in } \boldsymbol{R}\times(0,\infty) \end{aligned} \tag{1.1}$$

with initial conditions

$$u(\cdot, 0) = u_0 \text{ and } v(\cdot, 0) = v_0 \text{ on } \boldsymbol{R}. \tag{1.2}$$

Here, $m>1$ and $n\geq 1$ are real numbers. Throughout this paper, we assume that $m>1$ and $n\geq 1$.

By [10], the following properties are shown:

When the reaction arises among some reactions, for each reactant the equation for reaction-diffusion takes the form

$$\frac{\partial C}{\partial t} = \operatorname{div} D \operatorname{grad} C + q',$$

where C is the concentration, D is the diffusion coefficient and q' is the amount of material formed through chemical reactions per unit volume per unit time. When a reaction arises among n molecules of a substance A and n molecules of a substance B and does not reverse, that is to say, when the reaction is written as

$$nA + nB \mp \text{product},$$

then q' of both equations for A and B are proportional to $-C_A^n C_B^n$, where C_A and C_B are the concentrations of the substances A and B, respectively. That is to say, the concentrations C_A and C_B satisfy the equation

$$\begin{aligned} \frac{\partial C_A}{\partial t} &= \operatorname{div} D_A \operatorname{grad} C_A - kC_A^n C_B^n \\ \frac{\partial C_B}{\partial t} &= \operatorname{div} D_B \operatorname{grad} C_B - kC_A^n C_B^n, \end{aligned} \tag{1.3}$$