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Osaka J. Math.
29 (1992), 809–836

REPRESENTATION OF THE SCATTERING KERNEL FOR THE ELASTIC WAVE EQUATION AND SINGULARITIES OF THE BACK-SCATTERING

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(Received September 30, 1991)

1. Introduction and main results

By Yamamoto [15], Shibata and Soga [7], etc., we know that we can construct the scattering theory for the elastic wave equation corresponding to the theory for the scalar-valued wave equation formulated by Lax and Phillips [3, 4]. Employing Lax and Phillips' theory, Majda [5] obtained a representation of the scattering kernel (operator), which was very useful for investigation on the inverse scattering problems (cf. Majda [5], Soga [8, 10], etc.). In the present paper we shall give the similar representation of the scattering kernel for the elastic wave equation considered in Shibata and Soga [7], and examine the singular support of that kernel.

Let Ω be an exterior domain in $\boldsymbol{R}^n_x$ $(x=(x_1, \cdots, x_n))$ whose boundary $\partial\Omega$ is a compact C^∞ hypersurface, and consider the elastic wave equation

$$(1.1)\qquad \begin{cases} (\partial_t^2-\sum_{i,j=1}^n a_{ij}\partial_{x_i}\partial_{x_j})u(t,x)=0 & \text{in} \quad \boldsymbol{R}\times\Omega, \\ Bu(t,x)=0 \quad \text{on} \quad \boldsymbol{R}\times\partial\Omega, \\ u(0,x)=f_1(x), \quad \partial_t u(0,x)=f_2(x) & \text{on} \quad \Omega. \end{cases}$$

Here, $u={}^t(u_1, \cdots, u_n)$ is the displacement vector, a_{ij} are constant $n\times n$ matrices whose (p, q)-components a_{ipjq} satisfy

$$(\text{A}.1)\qquad a_{ipjq}=a_{pijq}=a_{jqip}, \quad i, j, p, q=1, 2, \cdots, n,$$

$$(\text{A}.2)\qquad \sum_{i,p,j,q=1}^n a_{ipjq}\varepsilon_{jq}\bar{\varepsilon}_{ip} \geqq \delta\sum_{i,p=1}^n |\varepsilon_{ip}|^2 \text{ for every symmetric matrices } (\varepsilon_{ij}),$$

and the boundary operator B is of the form

$$Bu=u|_{\partial\Omega} \quad \text{or} \quad \sum_{i,j=1}^n \nu_i(x)\, a_{ij}\partial_{x_j}u|_{\partial\Omega}\,,$$

where $\nu=(\nu_1, \cdots, \nu_n)$ is the unit outer vector normal to $\partial\Omega$. We denote by $U(t)$ the mapping: $f=(f_1, f_2)\to(u(t, \cdot), \partial_t u(t, \cdot))$ associated with (1.1), and by $U_0(t)$