Shiga, T.
Osaka J. Math.
29 (1992), 789-807

ERGODIC THEOREMS AND EXPONENTIAL DECAY OF SAMPLE PATHS FOR CERTAIN INTERACTING DIFFUSION SYSTEMS

Dedicated to Professor T. Watanabe on occasion of his 60th birthday

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(Received October 16, 1991)

1. Introduction and main results

Let S be a countable space. In the present paper we treat a class of diffusion processes taking values in a suitable subspace of R^S, which are governed by the following stochastic differential equation (SDE):

$$dx_i(t) = \sum_{j\in S} A_{ij}x_j(t)dt + a(x_i(t))dB_i(t), \quad i\in S, \tag{1.1}$$

where $\{B_i(t)\}_{i\in S}$ is an independent system of one-dimensional standard $\{\mathcal{F}_t\}$-Brownian motions defined on a complete probability space with filtration $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$.

We here assume

(1.2) $A=(A_{ij})$ is an $S\times S$ real matrix satisfying that $A_{ij}\geqq 0$ for $i\neq j$, $-A_{ii}=\sum_{j\neq i} A_{ij}<\infty$, and $\sup_{i\in S}|A_{ii}|<\infty$,

(1.3) $a(u)$: $R\to R$ is a locally 1/2-Hölder continuous function satisfying a linear growth condition: for some $C>0$,

$$|a(u)|\leq C(1+|u|) \quad \text{for } u\in R.$$

The diffusion models described by the SDE (1.1) arise in various fields such as mathematical biology and statistical physics. We here list several examples.

Example 1. (Stepping stone model with random drift [10])

$$a(u) = \begin{cases} \sqrt{u(1-u)} & \text{for } 0\leq u\leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Example 2. (Stepping stone model with radom selection [8])

$$a(u) = \begin{cases} u(1-u) & \text{for } 0\leq u\leq 1 \\ 0 & \text{otherwise.} \end{cases}$$