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LIFSCHITZ TAILS FOR RANDOM SCHRÖDINGER OPERATORS ON NESTED FRACTALS

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1. Introduction

The nested fractal introduced by Lindstrøm [7] is a certain class of fractal possessing finite ramifiedness and some symmetry. The Sierpinski gasket, the snowflake fractal and the Pentakun are members of this class (see [6], [7]). In this paper, we are concerned with two types of random operators on nested fractals; *the Laplacian with Poisson obstacles* and *the random Schrödinger operator* both formulated presently.

Let (Ψ, E) be a unit nested fractal in R^D constructed by a family of α-similitudes $\Psi=\{\Psi_0, \cdots, \Psi_{N-1}\}$ with $\Psi_0(x)=\alpha^{-1}x$, $\alpha>1$ (see Definition (2.1) and (2.2)). We then consider the expanded nested fractals defined by $E^{\langle m\rangle}=\alpha^m E$ and $E^{\langle\infty\rangle}=\bigcup_m E^{\langle m\rangle}$. By the Laplacian Δ, we mean the generator of Lindstrøm's Brownian motion of E ([7]). The associated Dirichlet form has been identified by Kusuoka [5]. We formulate our random operators by perturbing the corresponding Dirichlet form $(\mathcal{F}^{\langle m\rangle}, \mathcal{E}^{\langle m\rangle})$ on $L^2(E^{\langle m\rangle}; \mu)$, where μ is a $\log N/\log\alpha$-dimensional Hausdorff measure on $E^{\langle\infty\rangle}$ with $\mu(E)=1$.

Let $\mathcal{N}_\omega$ be the support of the Poisson random measure on $E^{\langle\infty\rangle}$ with the intensity measure $\nu\mu$ (ν is a positive constant) and let $\Delta_\omega^{\langle m\rangle}$ be the self-adjoint operator on $L^2(E^{\langle m\rangle}; \mu)$ associated with the Dirichlet form $(\mathcal{F}_\omega^{\langle m\rangle}, \mathcal{E}^{\langle m\rangle})$, where

$$\mathcal{F}_\omega^{\langle m\rangle} = \{f\in\mathcal{F}^{\langle m\rangle}; f(p)=0, p\in\mathcal{N}_\omega\cap E^{\langle m\rangle}\}.$$

$\Delta_\omega^{\langle m\rangle}$ is called the Laplacian with Poisson obstacles.

For another type of random operator, we first introduce a probability space $(\hat{\Omega}, \hat{\Sigma}, \hat{P})$ on the set $\hat{\Omega}$ of positive Radon measures on $E^{\langle\infty\rangle}$ so that restrictions of each measure to unit fractals constituting $E^{\langle\infty\rangle}$ behave as independent, identically distributed random variables (see (3.8)). The random Schrödinger operator is by definition the self-adjoint operator $H_{\hat{\omega}}^{\langle m\rangle}$ on $L^2(E^{\langle m\rangle}; \mu)$ associated with the Dirichlet form $(\mathcal{E}_{\hat{\omega}}^{\langle m\rangle}, \mathcal{F}^{\langle m\rangle})$, where

$$\mathcal{E}_{\hat{\omega}}^{\langle m\rangle}(u, v) = \mathcal{E}^{\langle m\rangle}(u, v)+\int_{E^{\langle m\rangle}} u(x)v(x)\hat{\omega}(dx) \quad \text{for } \hat{\omega}\in\hat{\Omega}.$$

The spectrum of $-\Delta_\omega^{\langle m\rangle}$ consists only of non-negative eigenvalues with