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STABLE HOMOTOPY TYPES OF STUNTED LENS SPACES MOD 4

Dedicated to Professor Hideki Ozeki on his 60th birthday

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1. Introduction

Let $L^n(q)=S^{2n+1}/(\boldsymbol{Z}/q)$ be the $(2n+1)$-dimensional standard lens space mod q. As defined in [8], we set

$$(1.1)\qquad \begin{aligned} L_q^{2n+1} &= L^n(q)\,, \\ L_q^{2n} &= \{[z_0, \cdots, z_n]\in L^n(q)\,|\, z_n \text{ is real}\geqq 0\}\ . \end{aligned}$$

The stable homotopy types (S-types) of stunted lens spaces L_q^m/L_q^n have been studied by several authors (e.g. [7], [8], [9], [10], [11] and [12]). For the case $q=2$, D.M. Davis and M. Mahowald have completed the classification of the stable homotopy types of stunted real projective spaces in [7]. Their result shows that we can use structures of J-groups of suspensions of stunted real projective spaces to obtain the necessary conditions for stunted real projective spaces $RP(m)/RP(n)$ and $RP(m+t)/RP(n+t)$ to have the same stable homotopy type as follows: if $RP(m)/RP(n)$ and $RP(m+t)/RP(n+t)$ have the same stable homotopy type, then there exists a non-negative integer N such that

$$\tilde{J}(S^j(RP(m)/RP(n))) \cong \tilde{J}(S^{j-t}(RP(m+t)/RP(n+t)))$$

for each integer j with $j\geqq N$ (see [13]). For the case where q is an odd prime, T. Kobayashi has obtained some necessary conditions for stunted lens spaces L_q^m/L_q^n and L_q^{m+t}/L_q^{n+t} to have the same stable homotopy type (cf. [10]). The conditions are also sufficient if $k=[m/2]-[(n+1)/2]\not\equiv 0 \pmod{(q-1)}$ or $n+1\equiv 0 \pmod{2q^{[k/(q-1)]}}$. We can use structures of J-groups of suspensions of stunted lens spaces mod q to obtain the conditions (see [14]). The object of this paper is to study the stable homotopy types of stunted lens spaces L_q^m/L_q^n for $q=4$ or 8.

In order to state our results, we prepare functions h_1, h_2, α, β_1, β_2 and γ_1 defined by

$$(1.2)\qquad h_1(k) = \begin{cases} [k/4]+[(k+7)/8]+[(k+4)/8] & (k\geqq 2) \\ 0 & (1\geqq k\geqq 0)\,. \end{cases}$$