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ON DIJKGRAAF-WITTEN INVARIANT FOR 3-MANIFOLDS

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1. Introduction

In 1990 Dijkgraaf and Witten [4] introduced a method of constructing an invariant of 3-manifolds using a finite gauge group G. For a closed oriented 3-manifold M, the Dijkgraaf-Witten invariant is given by the following formula:

$$Z(M)=\frac{1}{|G|}\sum_{\gamma\in\mathrm{Hom}(\pi_1(M),G)}\langle\gamma^*\alpha,[M]\rangle\,.$$

Here γ is a continuous map from a closed 3-manifold M to the classifying space BG of G, α is a cohomology class of $H^3(BG, U(1))$, γ^* is a map from $H^3(BG, U(1))$ to $H^3(M, U(1))$ induced from γ and $[M]$ is the fundamental class of M. However, in the case where M has a boundary, such a formulation can not be done, because the fundamental class $[M]$ is not defined for a manifold with boundary. To extend the definition of $Z(M)$ to a 3-manifold with boundary, they reduced the topological action $\langle\gamma^*\alpha, [M]\rangle$ to a lattice gauge theory. Furthermore Dijkgraaf and Witten asserted that their construction for a 3-manifold with boundary gives an example of a topological quantum field theory.

In this paper, we formulate an invariant of 3-manifolds possibly with boundary introduced by Dijkgraaf and Witten using a triangulation and prove its topological invariance in a rigorous way. Once given a finite group G and a 3-cocycle $\alpha\in Z^3(BG, U(1))$, the Dijkgraaf-Witten invariant is defined combinatorially. Throughout this paper, our target manifolds are compact oriented 3-manifolds with boundary or without boundary.

By a colour of M, we mean a map assigning an element of G to each oriented edge of a triangulated compact oriented 3-manifold M under some condition (See §2, for more precise definition). We call a map obtained from a colour of M by restricting it, to the oriented edges in ∂M a colour of ∂M. For a colour τ of ∂M, by $\mathrm{Col}(M, \tau)$ we denote the set of all colours of M which are equal to τ, when restricted to ∂M. Having a colour φ of M we associate with each 3-simplex σ of M a complex number $W(\sigma, \varphi)\in U(1)$ using α. We