Kanenobu, T.
Osaka J. Math.
29 (1992), 635–651

GENUS AND KAUFFMAN POLYNOMIAL OF A 2-BRIDGE KNOT

TAIZO KANENOBU*

(Received September 4, 1991)

In [6, Theorem 6], arbitrarily many 2-bridge knots sharing the same Jones polynomial are constructed. The construction is as follows (See Example 2.): Two 2-bridge knots 10_{22} and 10_{35} [15, Table] are obtained as symmetric skew unions [11] of 5_2. They have the same Jones polynomial [4] but have distinct genera, and so have distinct Alexander polynomials [3,14]. From these knots, we get four 2-bridge knots as symmetric skew unions. Continuing this construction, we have 2^N distinct 2-bridge knots with the same Jones polynomial for any positive integer N. The question is whether the Alexander polynomials, or genera, of these 2-bridge knots are mutually distinct or not.

In [7, Theorem 5], we constructed a pair of 2-bridge knots which have the same Kauffman polynomial and so have the same Jones polynomial, but have distinct Alexander polynomials. In fact, in the set of all the 2-bridge knots through 22 crossings, there are 239 pairs sharing the same Kauffman polynomial, among which 58 pairs also share the same homfly polynomial and the rest have distinct Alexander polynomials [9]. Also in [8], we constructed arbitrarily many skein equivalent 2-bridge knots with the same Kauffman polynomial, and so they have the same homfly, Jones and Alexander polynomials. We refer [13] for the definition of the skein equivalence and the homfly polynomial, and [10] for the Kauffman and L polynomials and the writhe. In this paper, we prove:

Theorem, *For any positive integer N, there exist N 2-bridge knots with the same Kauffman polynomial but distinct genera.*

Also for 2-bridge links, we can construct a similar example. In Section 1, we give a geometric algorithm ((8) and Proposition 2) deforming a 2-bridge knot or link in the form of a 4-plat or Conway's normal form $C(a_1, a_2, \cdots, a_n)$ [2] into the special form $D(b_1, b_2, \cdots, b_{2g+\mu-1})$, where g is the genus and μ the number of the components. Throughout the deformation, the 2-bridge knot or link

* This work was supported in part by Grant-in-Aid for Encouragement of Young Scientist (No. 02740046), Ministry of Education, Science and Culture.