

## CUT-AND-PASTES OF INCOMPRESSIBLE SURFACES IN 3-MANIFOLDS

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### 1. Introduction

Let  $M$  be a compact orientable 3-manifold and  $F_1$  and  $F_2$  properly embedded surfaces in  $M$ . If  $F_1$  and  $F_2$  intersect transversely, then by cutting  $F_1$  and  $F_2$  along the intersection and regluing them in a different way, we obtain another embedded surface in  $M$ .

DEFINITION. Let  $F_1$  and  $F_2$  be orientable surfaces properly embedded in  $M$  intersecting transversely. A *cut-and-paste (CP) operation* on a component  $C$  of  $F_1 \cap F_2$  is the following operation in a regular neighborhood of  $C$ ,  $N(C)$ : Cut  $F_1$  and  $F_2$  on  $C$  and reglue them in a different way. See Figure 1.1.

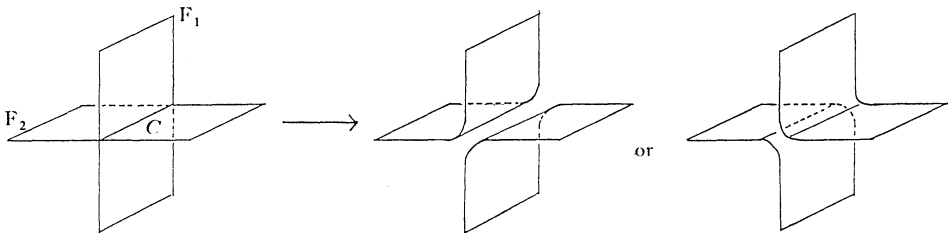


Fig 1.1.

Note that there are two choices in regluing. When we apply a CP operation on each component of  $F_1 \cap F_2$ , we obtain an embedded surface  $F$  in  $M$ . We say that  $F$  is obtained from  $F_1$  and  $F_2$  by a (way of) CP operation.

Suppose that both  $F_1$  and  $F_2$  are incompressible. In general, a surface which is obtained from  $F_1$  and  $F_2$  by a CP operation is possibly compressible. But we can prove that in certain cases there is a CP operation which yields an

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