Sumi, T.
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Z/kZ-FINITENESS FOR CERTAIN S^1-SPACES

TOSHIO SUMI

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Introduction

Let G-$\mathcal{FDCW}$ denote the category of G-spaces having the G-homotopy type of a finitely dominated G-CW complex for a compact Lie group G. Lück [8] has introduced a functor Wa^G from G-$\mathcal{FDCW}$ into the category of abelian groups and has realized the equivariant finiteness obstriction as the element $w^G(X)$ in $Wa^G(X)$. That is, a finitely dominated G-CW complex X is G-homotopy equivalent to a finite G-CW complex if and only if $w^G(X)=0$. When G is the trivial group, there is an isomorphism from $Wa^G(X)$ to the reduced projective group $\tilde{K}_0(\boldsymbol{Z}[\pi_1(X)])$ which sends the element $w^G(X)$ to the Wall's finiteness obstruction ([14]).

Anderson [1] and Ehrlich [4] have studied a sufficient condition for $w^{\{1\}}(E)=0$ for some fibration $E\to B$ with fiber S^1. Munkholm, Pedresen [11], Lück [6, 7, 9] and others have studied the transfer map $\tilde{K}_0(\boldsymbol{Z}[\pi_1(B)])\to\tilde{K}_0(\boldsymbol{Z}[\pi_1(E)])$. The purpose of this paper is to get a sufficient condition for $w^L(X)=0$ for a S^1-space X and a finite cyclic group L.

We call G-maps f_0: $Y_0\to X$ and f_0: $Y_4\to X$ equivalent if there exists a commutative diagram

$$\begin{tikzcd}
Y_0 \arrow[r, hook] \arrow[drr, "f_0"'] & Y_1 \arrow[r, "\overset{\simeq}{\mathrm{G}}"] \arrow[dr, "f_1"] & Y_2 \arrow[r, "\overset{\simeq}{\mathrm{G}}"] \arrow[d, "f_2"] & Y_3 \arrow[dl, "f_3"'] & Y_4 \arrow[l] \arrow[dll, "f_4"] \\
 & & X & &
\end{tikzcd}$$

such that (Y_1, Y_0) and (Y_3, Y_4) are relatively finite G-CW complexes, and $Y_1\to Y_2$ and $Y_3\to Y_2$ are G-homotopy equivalences. The group $Wa^G(X)$ consists of equivalence classes $[f\colon Y\to X]$ of the set of G-maps $f\colon Y\to X$ with Y finitely dominated and $w^G(X)$ is the equivalence class containing the identity 1_X of X. The additive structure on $Wa^G(X)$ is given by a disjoint sum:

$$[f\colon Y\to X]+[g\colon Z\to X]=[f\amalg g\colon Y\amalg Z\to X]$$