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LINEARITY OF HOMOTOPY REPRESENTATIONS

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0. Introduction

In the theory of transformation groups, it is an important problem to distinguish whether or not a group action is linear. In this paper we would like to consider linearity of homotopy representations of finite groups in the G-homotopy category.

Since the study of homotopy representations of finite groups G due to tom Dieck-Petrie [6], it is known that there exist many homotopy representations which are not linear (i.e., not G-homotopy equivalent to linear G-spheres). On the other hand, in [5], tom Dieck proved that any homotopy representation of a cyclic p-group C_{p^m} is linear under a restricted situation. For its proof, tom Dieck used the stable theory of homotopy representations.

We first consider the following problem under the general setting. We use the unstable theory of homotopy representations.

Problem. *When is a homotopy representation of G linear?*

If a homotopy representation is linear, its dimension function must be linear at least. Therefore we mainly discuss homotopy representations with linear diemsnion functions. (For the linearity of dimension functions, see [1], [2], [3], [6].)

In Section 1 we recall some definitions and well-known results, in particular, the unstable Picard group $\mathrm{Pic}(G;\underline{n})$ and Laitinen's invariant ([7]), which are the main tools in this paper.

In Section 2 we introduce subgroups $jO(G;\underline{n})$ and $\mathrm{Pic}^f(G;\underline{n})$ of $\mathrm{Pic}(G;\underline{n})$ for any linear dimension function $\underline{n}$, and put

$$LH^\infty(G;\underline{n}) = \mathrm{Pic}(G;\underline{n})/jO(G;\underline{n}),$$
$$LH(G;\underline{n}) = \mathrm{Pic}^f(G;\underline{n})/jO(G;\underline{n}).$$

For any homotopy representation X with dimension function $\underline{n}$, we define a G-homotopy invariant $l(X)$ in $LH^\infty(G;\underline{n})$ by using Laitinen's invariant, and

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