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HOMOLOGY BOUNDARY LINKS AND FUSION CONSTRUCTIONS

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Introduction

It is well-known [6] that a smooth link of spheres $L^n \subset S^{n+2}$ is trivial if and only if

i) the link group $\pi=\pi_1(S^{n+2}\backslash L)$ is free on the set of meridians of L, and

ii) the homotopy groups $\pi_j(S^{n+2}\backslash L)$ are trivial for $2\leq j\leq[(n+1)/2]$,

at least when $n\neq 2$. It is of considerable interest to study classes of links for which the link group satisfies weaker "freeness" conditions than i). The following three consecutively enlarged classes turn out to be of special importance:

(I) *Boundary links*. These are links whose components bound disjoint oriented manifolds in S^{n+2} (Seifert surfaces). Equivalently, there is an epimorphism $\pi\to F$ (F=free group of rank equal to the number of components of L), which maps meridians to generators.

(II) *Homology boundary links* (*HBLs*). Here we drop the condition on meridians. A geometric interpretation in terms of "singular" Seifert surfaces is known (compare [15]).

(III) *Sublinks of homology boundary links* (*SHB-links*). This class arises since class (II) is not closed with respect to sublinks.

Interest in these classes comes mostly from the study of link concordance. In higher dimensions a classification of boundary links up to boundary link concordance is known (compare for example [9]). Recently T. Cochran and J. Levine proved that each *HBL* is *concordant* to a fusion of a boundary link [3]. Roughly, a *fusion* of an r-component link is an $(r-j)$-component link, which is formed by attaching j 1-handles (bands) to the link. The question whether in the last statement *concordance* can be replaced by *isotopy* is in fact related to the Andrews Curtis Conjecture (see [3]). Cochran and Levine define an obstruction for a *HBL* to be a boundary link, the *pattern*, an isotopy invariant, which in the author's opinion has not yet been studied adequately. In 1989, T. Cochran and K. Orr proved the result, surprising to the experts, that there are *HBL*s, which are not even *concordant* to boundary links [4]. Their examples arise from a "completed" fusion construction, which preserves the number of components: