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UNIQUENESS OF POSITIVE SOLUTIONS OF THE HEAT EQUATION

Dedicated to Professor Masanori Kishi on his 60th birthday

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1. Introduction

Let D be an unbounded domain in $\boldsymbol{R}^2$ and u a nonnegative solution of the heat equation on D. We consider the property (U) for D:

$$\text{(U):} \qquad u=0 \quad \text{on} \quad \partial_p D \Rightarrow u=0 \quad \text{on}\, D\,,$$

where $\partial_p D$ is the parabolic boundary of D. In the case of $D=\boldsymbol{R}\times(0, T)$ or $(0, \infty)\times(0, T)$, it is known that the property (U) holds (see [6]).

In this paper, by using a special form of the boundary Harnack principle, we shall show the following generalization.

Theorem. *For $T\in(-\infty, \infty]$ and an upper semicontinous function φ on $\boldsymbol{R}$, we set*

$$D(\varphi, T)=\{(x, t);\, \varphi(x)<t<T\}\ .$$

If φ is bounded below, then the property (U) *holds for $D(\varphi, T)$.*

By Theorem we obtain the following

Corollary. *Let φ, T and $D(\varphi, T)$ be as in Theorem, and assume that φ is bounded below. Let u, v be positive solutions of the heat equation on $D(\varphi, T)$. If $u-v$ vanishes continuously on $\partial_p D(\varphi, T)$, then $u=v$ on $D(\varphi, T)$.*

On the other hand, for $D=\{(x, t);\, mt<x\}$, the property (U) does not hold (see Lemma 7, Proposition 2). By using the Appell transformation, we shall show that this is critical (see Proposition 1).

2. Preliminaries

For a domain D in $\boldsymbol{R}^2$ we denote by $\partial_p D$ the set of $(x, t)\in\partial D$ (the boundary of D) satisfying $V\cap D\cap(\boldsymbol{R}\times(t, \infty))\neq\emptyset$ for every neighborhood V of (x, t) and call it the parabolic boundary of D. For $X=(x, t)\in D$, we denote by ω_D^X