Engler, H. and Hetzer, G.
Osaka J. Math.
29 (1992), 471–481

CONVERGENCE TO EQUILIBRIA FOR A CLASS OF REACTION-DIFFUSION SYSTEMS

HANS ENGLER AND GEORG HETZER

(Received June 1, 1991)

1. Introduction

An important question in the study of reaction-diffusion systems is the identification of all steady states and the classification of their stability. In this note, we give a contribution to this question for a class of systems with two diffusing and reacting components.

$$\partial_t u_i - d_i \Delta u_i = f_i(u_1, u_2) \qquad (i = 1, 2) \tag{1.1}$$

in some cylindrical time-space domain $\Omega \times (0, \infty) \subset \boldsymbol{R}^{N+1}$, together with zero Neumann boundary data on the lateral boundary $\partial\Omega \times (0, \infty)$ and suitable initial data on $\Omega \times \{0\}$. Then all equilibria of the associated system of ordinary differential equations

$$\dot{y}_i = f_i(y_1, y_2) \qquad (i = 1, 2) \tag{1.2}$$

are also spatially constant steady states of (1.1). Consider now the special case

$$f_i(u_1, u_2) = u_i(a_{i0} - a_{i1}u_1 - a_{i2}u_2) \tag{1.3}$$

with $a_{ij} > 0$ for all i, j. In this case, it turns out that the unique positive equilibrium of (1.2) (if it exists) is globally asymptotically stable for (1.1) if and only if it is so for (1.2). This will be the case if and only if

$$\frac{a_{11}}{a_{21}} > \frac{a_{10}}{a_{20}} > \frac{a_{12}}{a_{22}}\,; \tag{1.4}$$

see [12] for a detailed study and for results on other possible combinations of coefficients.

Note that in this case $\dfrac{\partial f_i}{\partial u_j} \leq 0$ for $i \neq j$; such a vector field is called *competitive*. On the other hand, it was shown in [11] that for competitive vector

* Supported by a grant from the National Science Foundation and by the Deutsche Forschungsgemeinschaft (SFB 256).