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A REMARK ON FINITE POINT TRANSITIVE AFFINE PLANES WITH TWO ORBITS ON l_∞

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In this note for the most part we shall use the notations of [1]. Let $\mathcal{P}=\pi\cup l_\infty$ be the projective extension of an affine plane and G a collineation group of $\mathcal{P}$. If p is a point of $\mathcal{P}$ and l is a line of $\mathcal{P}$, then $G(p, l)$ is the subgroup G consisting of all perspectivities in G with center p and axis l. If m is a line of $\mathcal{P}$, then $G(m, m)$ is the subgroup of all elations in G with axis m.

In [3] the author proved the following theorem on Kallaher's conjecture (See [1]).

Theorem 1. *Let π be a finite affine plane of order n with a collineation group G which is transitive on the affine points of π. Suppose that G has two orbits of length 2 and n-1 on l_∞. Then π is a translaiton plane and the group G contains the group of translations of π, except in the following case:*

(*) $|G(l_\infty, l_\infty)|=n=2^m$ *for some* $m\geq 1$, $G(p_1, l_\infty)=G(p_2, l_\infty)=1$ *and* $|G(p, l_\infty)|=2$ *for all* $p\in l_\infty-\{p_1, p_2\}$, *where* $\{p_1, p_2\}$ *is a G-orbie of length 2 on* l_∞.

The case (*) actually occurs when π is a desarguesian plane of order 2. Maharjan [2] studied the planes with property (*) under the condition that $n\leq 4$. The purpose of this note is to prove the following.

Proposition 2. *Assume* (*). *Then $n=2$ and G is a cyclic group of order* 4.

Maharjan [2] proved the proposition under the condition that $n\leq 4$.

The proposition, together with Theorem 1, gives the following.

Theorem 3. *Let π be a finite affine plane of order n with a collineation group G which is transitive on the affine points of π. If G has two orbits of length 2 and $n-1$ on l_∞, then one of the following statements holds:*

(i) *The plane π is a tanslation plane and the group G contains the group of translations of π.*

(ii) $n=2$ *and G is a cyclic group of order* 4.

In the rest of the note, we prove Proposition 2. Set $T=G(l_\infty, l_\infty)$ and $L=G_{P_1,P_2}$. Then $|G:L|=2$. Let O be an affine point of π. Set $l=P_1 O$. Sup-