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ON ALGEBRAS WHICH RESEMBLE THE LOCAL WEYL ALGEBRA

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1. Introduction

Let K be an algebraically closed field of characteristic zero and let $\hat{\mathcal{O}}_n(K)=K[[x_1, \cdots, x_n]]$ be the formal power series ring over K in n variables. According to Björk [1], we denote by $\hat{D}_n(K)$ the subring of $\mathrm{End}_K(\hat{\mathcal{O}}_n(K))$ generated over K by the left multiplications by elements of $\hat{\mathcal{O}}_n(K)$ and partial differentials $\partial_i=\partial/\partial x_i$,

$$\hat{D}_n(K) = \hat{\mathcal{O}}_n(K)\langle\partial_1, \cdots, \partial_n\rangle$$

where $\partial_i x_j - x_j \partial_i = \delta_{ij}$ (Kronecker's delta) and $\partial_i \partial_j = \partial_j \partial_i$. The ring $\hat{D}_n(K)$, called the *local Weyl algebra*, has the Σ-filtration $\{\Sigma_v\}_{v\geq 0}$ such that $\Sigma_0=\hat{\mathcal{O}}_n(K)$ and $\Sigma_v=\{\Sigma_\alpha f_\alpha \partial^\alpha; f_\alpha \in \hat{\mathcal{O}}_n(K)$ and $\partial^\alpha=\partial_1^{\alpha_1}\cdots\partial_n^{\alpha_n}$ with $|\alpha|=\alpha_1+\cdots+\alpha_n\leq v\}$ and that the associated graded ring $\mathrm{gr}_\Gamma(\hat{D}_n(K))$ is a polynomial ring over $\hat{\mathcal{O}}_n(K)$ in n variables. Moreover, $\hat{D}_n(K)$ has weak global dimension n, i.e., $\mathrm{w.gl.dim}(\hat{D}_n(K)) = n$.

These are ring-theoretic, algebraic properties which the local Weyl algebra $\hat{D}_n(K)$ has. In the present article, we consider whether or not these properties are sufficient to characterize the ring $\hat{D}_n(K)$. For this purpose, we introduce the notion of pre-W-algebra and W-algebra (see below for the definition) and show that a W-algebra, which satisfies the above-listed properties $\hat{D}_n(K)$ has and one additional condition, i.e., $L=\Sigma_1/\Sigma_0$ is essentially abelian, is realized as a subalgebra of some $\hat{D}_n(K)$. After all, we are successful only in the case $n=1$. We are, however, convinced that our approach of computing the weak global dimension of a W-algebra will be useful to study locally a vector field at a smooth point on an algebraic variety.

We employ the terminology and notation in [1].

2. Structure theorems

To simplify the notation, we denote $\hat{\mathcal{O}}_n(K)$ by R. Let A be a (not necessarily commutative) K-algebra containing R generated by finitely many elements