Kim, S.S.
Osaka J. Math.
29 (1992), 387–392

DIVISIBILITY CONDITIONS ON SIGNATURES OF FIXED POINT SETS

SUNG SOOK KIM

(Received June 12, 1991)

Let G denote the cyclic group of order p, where p is an odd prime. In [6], we constructed a smooth G-action on some $\boldsymbol{Z}_q$-homology sphere such that the fixed point set is a closed connected $4r$-dimensional manifold with nonzero Pontryagin numbers, where q is an odd prime distinct from p.

In this paper we take some preliminary steps towards studying the divisibility conditions on the characteristic numbers of the fixed point set of a G-action on a $\boldsymbol{Z}_q$-homology sphere. One reason for interest in this topic is that the image of the fixed point set of a G-action on a $\boldsymbol{Z}_q$-homology sphere in Ω_*^{SO}/torsion is completely determined by these divisibility conditions. For some time it has been known that nontrivial conditions appear (compare [5]; [2]). Perhaps the simplest divisibility condition involves the signature of the fixed point set. If G acts smoothly and preserving orientation on a closed oriented even dimensional $\boldsymbol{Z}_q$-homology sphere, then the signature of the fixed point set must be even because the Euler characteristic number is 2 by the Lefschetz fixed point theorem and the signature and Euler characteristic number of a closed oriented manifold are always congruent modulo 2.

Our first theorem is the following, which is proved by using the G-signature theorem.

Theorem 1. *Let X be a smooth closed oriented manifold of even dimension such that $H^{(\dim X)/2}(X;\boldsymbol{Q})=0$. If the fixed point set F of a smooth G-action on X is 4-dimensional, then*

$$4\,|\,\mathrm{Sign}\,F\,,\ \textit{when}\ p>3\quad \textit{and}$$
$$16\,|\,\mathrm{Sign}\,F\,,\ \textit{when}\ p=3\,.$$

Following Kawakubo [5] we say that a smooth G-action is regular if the normal G vector bundle of the fixed point set is decomposed by only one eigen-bundle; i.e. it is of the form $\xi_m\otimes t^m$ for some m $(1\leq m\leq \frac{p-1}{2})$, where ξ_m is a complex vector bundle with trivial G-action and t^m is the complex 1-dimensional

The author was partially supported by KOSEF Grant 913-0103-015-2.