Yosimura, Z.
Osaka J. Math.
29 (1992), 361–385

THE K_*-LOCALIZATIONS OF WOOD AND ANDERSON SPECTRA AND THE REAL PROJECTIVE SPACES

ZEN-ICHI YOSIMURA

(Received May 9, 1991)

0. Introduction

Let E be an associative ring spectrum with unit. For any CW-spectra X and Y we say that X is *quasi E_*-equivalent* to Y (see [15] or [16]) if there exists a map $f: Y \to E_\wedge X$ such that the composite map $(\mu_\wedge 1)(1_\wedge f): E_\wedge Y \to E_\wedge X$ is an equivalence where $\mu: E_\wedge E \to E$ denotes the multiplication of E. Let KO, KU and KT be the real, the complex and the self-conjugate K-spectrum respectively (see [3] or [7]). It is known that there is no difference among the KO_*-, KU_*- and KT_*-localizations ([11], [5] or [13]). So we denote by S_K the K_*-localization of the sphere spectrum $S=\Sigma^0$. These spectra KO, KU, KT and S_K are all associative ring spectra with unit.

In [15] we studied the quasi K_*-equivalences, especially the quasi KO_*-equivalence, and in [16] and [17] we determined the quasi KO_*-types of the real projective spaces RP^n and the stunted real projective spaces RP^n/RP^m. In this note we will be interested in the quasi S_{K*}-equivalence in advance of the quasi KO_*-equivalence. According to the smashing theorem [6, Corollary 4.7] (or [13]), for any CW-spectrum X the smash product $S_{K\wedge}X$ is actually the K_*-localization of X. Hence we notice that two CW-spectra X and Y have the same K_*-local type if and only if X is quasi S_{K*}-equivalent to Y.

For any map $f: X \to Y$ its cofiber is usually denoted by $C(f)$. Let $\eta: \Sigma^1 \to \Sigma^0$ be the stable Hopf map of order 2. The KO-homologies of the cofibers $C(\eta)$ and $C(\eta^2)$ are well known as follows: $KO_iC(\eta) \cong \pi_i KU \cong Z$ or 0 according as i is even or odd, and $KO_iC(\eta^2) \cong \pi_i KT \cong Z$, $Z/2$, 0 or Z according as $i \equiv 0, 1, 2$ or 3 mod 4. A CW-spectrum X is said to be *a Wood spectrum* if it is quasi KO_*-equivalent to the cofiber $C(\eta)$, and *an Anderson spectrum* if it is quasi KO_*-equivalent to the cofiber $C(\eta^2)$ (see [12], [15] or [18]).

Let $\bar{\eta}: \Sigma^1 SZ/2 \to \Sigma^0$ and $\tilde{\eta}: \Sigma^2 \to SZ/2$ be an extension and a coextension of η with $\bar{\eta}i=\eta$ and $j\tilde{\eta}=\eta$, where $SZ/2$ denotes the Moore spectrum of type $Z/2$ constructed by the cofiber sequence $\Sigma^0 \xrightarrow{2} \Sigma^0 \xrightarrow{i} SZ/2 \xrightarrow{j} \Sigma^1$. Choose two maps $\bar{h}$: $\Sigma^3 SZ/2 \to C(\bar{\eta})$ and $\bar{k}$: $\Sigma^5 SZ/2 \to C(\bar{\eta})$ with $\bar{j}\bar{h}=\tilde{\eta}j$ and $\bar{j}\bar{k}=\tilde{\eta}\bar{\eta}$ where $\bar{j}$: $C(\bar{\eta}) \to \Sigma^2 SZ/2$ denotes the bottom cell collapsing. Using a fixed Adams' K_*-equiva-