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THE STRUCTURE OF THE COHOMOLOGY OF MORAVA STABILIZER ALGEBRA S(3)

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Introduction.

Let X be a space and let p be a prime number. The E_2-term of the Adams-Novikov spectral sequence associated with BP-theory at p converging to the p-localized homotopy group of X is given by $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*(X))$ ([1], [4]). This motivates to study $\mathrm{Ext}_{BP_*BP}(BP_*, M)$ for a BP_*BP-comodule M. If X is a finite complex, $BP_*(X)$ is a finitely presented BP_*-module ([3]). Recall ([1], [14]) that $BP_*=\boldsymbol{Z}_{(p)}[v_1, v_2, \cdots]$, $deg\ v_n=2(p^n-1)$, and I_n denotes an invariant prime ideal $(p, v_1, v_2, \cdots, v_{n-1})$ of BP_*. Landweber proved the following theorem.

Theorem ([6]). *Let M be a BP_*BP-comodule which is finitely presented as a BP_*-module. Then, M has a finite filtraiion by BP_*BP-subcomodules $0=M_0 \subset M_1 \subset \cdots \subset M_k=M$ such that for $1 \leq i \leq k$, M_i/M_{i-1} is isomorphic to BP_*/I_{n_i} for some $n_i \geq 0$ as a BP_*BP-comodule up to shifting degrees.*

By virtue of the above and a spectral sequence $E_2^{s,t}=\mathrm{Ext}_{BP_*BP}^{s+t}(BP_*, M_t/M_{t-1}) \Rightarrow \mathrm{Ext}_{BP_*BP}^{s+t}(BP_*, M)$ (See section 2) for M as above, we can relate $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_n)$ $(n=0, 1, 2, \cdots)$ with $\mathrm{Ext}_{BP_*BP}(BP_*, M)$. Hence it is necessary to know $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_n)$ before we study the general case.

For small n, $\mathrm{Ext}_{BP_*BP}(BP_*, BP/I_n)$ also has a geometric significance since there is a spectrum $V(n)$ whose BP-homology is isomorphic to BP_*/I_{n+1}, generalizing the Moore spectrum, if $p>2n$ and $n=0, 1, 2, 3$ ([2], [15]). Hence $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_{n+1})$ is the E_2-term of the Adams-Novikov spectral sequence converging to the homotopy group of $V(n)$. We note that $V(n)$ is a ring spectrum if $p>2n+2$ ([15]).

Since multiplication by v_n on BP_*/I_n is a BP_*BP-comodule homomorphism, $v_n^{-1}BP_*/I_n$ is a BP_*BP-comodule and $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_n)$ is a module over $\boldsymbol{F}_p[v_n]$ if $n>0$. In fact, $\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_n)$ is a graded commutative algebra and $\mathrm{Ext}_{BP_*BP}^0(BP_*, BP_*/I_n)$ is isomorphic to $\boldsymbol{F}_p[v_n]$ if $n>0$ ([5], [11]). Thus $v_n^{-1}\mathrm{Ext}_{BP_*BP}(BP_*, BP/I_n)$ makes sense and it is obviously isomorphic to $\mathrm{Ext}_{BP_*BP}(BP_*, v_n^{-1}BP_*/I_n)$.