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HOMOLOGOUS FIBRES AND TOTAL SPACES

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0. Introduction

In this note we consider fibrations of the form $F \to E \to B$ where all spaces involved have the homotopy type of pointed connected CW-complexes. Well-known work on the plus-construction (for algebraic K-theory, *et al*) reveals the following situation concerning when $E \to B$ induces an isomorphism of homology groups with trivial integer coefficients.

Theorem 0.1 [4]. *The following are equivalent.*
(i) *F is acyclic;*
(ii) *$H_*(E) \to H_*(B)$ is an isomorphism, and $\pi_1(B)$ acts trivially on $H_*(F)$.*

We focus here on a dual problem of when $H_*(F) \to H_*(E)$ can be an isomorphism. In general, mere acyclicity of B does not suffice, as evidenced by the following.

EXAMPLE 0.2. Let $Re \rightarrowtail Fr \twoheadrightarrow G$ be a free presentation of a finitely generated acyclic group G, with Fr of finite rank. By passing to classifying spaces we obtain a fibration as in the first sentence above. If G is non-trivial, then it is well-known that the rank of Re and $H_1(Re)$ exceeds that of Fr and $H_1(Fr)$ [13 I §3].

Here is the counterpart to Theorem 0.1.

Theorem 0.3 [8]. *The following are equivalent.*
(i) *B is acyclic, and $\pi_1(B)$ acts trivially on $H_*(F)$;*
(ii) *$H_*(F) \to H_*(E)$ is an isomorphism.*

However we shall now show how it is possible to remove the hypothesis of trivial fundamental group action (orientability) in favourable circumstances. We thereby derive assumptions under which acyclicity of B implies that $H_*(F) \to H_*(E)$ is an isomorphism for **all** fibrations involving the given F, E and B. The price is a further condition, either on B or on F. For the former ap-