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A BOUNDARY LINK IS TRIVIAL IF THE LUSTERNIK-SCHNIRELMANN CATEGORY OF ITS COMPLEMENT IS ONE

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1. Introduction

The Lusternik-Schnirelmann category cat(X) of a space X is the least integer n such that X can be covered by $n+1$ open subsets each of which is contractible to a point in X. In particular, cat(X) is a homotpoy type invariant and cat$(S^n)=1$. We know that $\pi_1(X)$ is a free group if X is a manifold and cat$(X)\leqq 1$ (cf. [2], [3] and [5]).

A locally flat knot (S^{n+2}, S^n) is topologically unknotted if and only if the category of its complement is one [10]. In fact, $S^{n+2}-S^n\simeq S^1$ if and only if cat$(S^{n+2}-S^n)=1$. We see also that a smooth knot (S^{n+2}, S^n) is unknotted if and only if cat$(S^{n+2}-S^n)=1$ when $n\neq 2$ ([7] for $n\geqq 4$, [15] for $n=3$ and [12] for $n=1$).

We will generalize this result to the smooth m-component links. A smooth (or locally flat) m-component link L stands for m smoothly (or locally flatly) embedded disjoint n-spheres $L_1\cup\cdots\cup L_m$ in S^{n+2}. A smooth (or locally flat) m-component link is called trivial if it bounds m smoothly (or locally flatly) embedded disjoint $(n+1)$-disks; boundary if it bounds a Seifert man ifold which consists of m disjoint compact smooth (or locally flat) $(n+1)$-submanifolds with connected boundary. Let $N_i=N(L_i)$ $(i=1, \cdots, m)$ be tubular neighborhoods of L_i which do not intersect each other. The $(n+2)$-dimensional compact manifold $E=S^{n+2}$- $\cup$ Int $N(L_i)$ with boundary $\partial E=\cup\,\partial N_i$ is called a link exterior and is homotopy equivalent to the link complement $S^{n+2}-L$.

In this paper we will show the following theorem by applying the unlinking criterion of boundary links due to Gutiérrez [6].

Theorem 1. *Let L be a smooth m-component boundary link in S^{n+2}. Suppose that $n\neq 2$. Then L is trivial if and only if* cat$(S^{n+2}-L)=1$.

If L is trivial, $S^{n+2}-L\simeq(\vee_m S^1)\vee(\vee_{m-1} S^{n+1})$. We have only to prove the if-part. On the other hand a classical link L is trivial if $\pi_1(S^3-L)$ is free by the loop theorem [12]. Since cat$(S^{n+2}-L)=1$ implies that $\pi_1(S^{n+2}-L)$ is free,