ADDITIVE FUNCTIONALS, NOWHERE RADON AND KATO CLASS SMOOTH MEASURES ASSOCIATED WITH DIRICHLET FORMS

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0. Introduction

The theory of regular Dirichlet forms $(E, \mathcal{F})$ associated with a locally compact separable metric space $\mathcal{X}$ and a positive Radon measure m s.t. supp$[m] = \mathcal{X}$ is a well developed subject, both from the potential analytic and the probabilistic point of view.   It has its origins in work by Beurling-Deny and was particularly pursued by Fukushima and Silverstein see e.g. [19], [27] and references therein.   It presents, at least in the symmetric case, a natural extension of the continuous functions framework of classical and axiomatic potential theory in the functional analytical (L^2-functions) direction, covering in particular a stochastic calculus for generators with coefficients which are not restricted to be functions (the associated processes need not be semimartingales).   This theory has turned out, in the last 15 years, to be particularly suited for applications in quantum theory, see e.g. [4], [5], [20], [1], [8].   In this field, but also in other contexts, see e.g. [2], there is the necessity of studying certain generalized functionals of the processes (of Feynman-Kac type), corresponding to singular perturbations of a given Dirichlet form (e.g. the one associated with the Laplacian over R^d).   This is discussed e.g. in [2], [10], [28], [29], [30], [16], [3], [1], [15], [11], [12], [23], [22] and references therein.   Many of the discussions have been concerned with functionals associated with measures in the so called Kato class (cfr. [9], [26]).   They are particular cases of smooth measures (in the sense of [19]) for the Dirichlet form associated with the Laplacian.   It is natural to ask oneselves what happens if one tries to carry through similar constructions using an *arbitrary* smooth measure associated with a general (regular) Dirichlet form.   In the present paper we initiate such a study.   We give results on the structure

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