Nishimura, T.
Osaka J. Math.
29 (1992), 233–245

EXTERIOR PRODUCT BUNDLE OVER COMPLEX ABSTRACT WIENER SPACE

TAKASHI NISHIMURA

(Received April 9, 1991)

1. Introduction

In this paper, we consider a *complex abstract Wiener space* (CAWS) (B,H,μ), that is a triplet of a complex separable Banach space B, a complex separable Hilbert space H which is densely and continuously imbedded in B and a Borel probability measure μ on B such that

$$\int_B \exp(\sqrt{-1}\,\mathrm{Re}_B\langle z, \varphi\rangle_{B^*})\,\mu(dz) = \exp(-\frac{1}{4}\|\varphi\|_{H^*}^2) \quad \text{for } \varphi\in B^*\subset H^*. \tag{1.1}$$

Moreover, we assume that a strictly positive self-adjoint operator A on H^* is given and $B^*\subset C^\infty(A)=\bigcap_{n=1}^{\infty}\mathrm{Dom}(A^n)$. Then we can define $D_Ap(z)=(\sqrt{A}\oplus\sqrt{\bar{A}})Dp(z)$ for $p\in\mathcal{P}(B\colon E)$, E-valued polynomial functional on B.

H-derivative D is a fundamental tool in Malliavin's calculs ([6]), but here we consider D_A instead of D, because we keep quantum field theoretical models in mind. In fact, $\frac{1}{2}D_A^*D_A=d\Gamma(A\oplus\bar{A})$, a free Hamiltonian for a complex Bose field (and its anti-particle field).

Following [3] and [4], we regard B as an infinite dimensional manifold with cotangent space $(H_R^*)^c$ on each $z\in B$. Consequently its exterior product bundle becomes $B\times\Lambda(H_R^*)^c$ and the space of its L^2-sections becomes $L^2(B, \mu\colon \Lambda(H_R^*)^c)$, i.e. the space of $\Lambda(H_R^*)^c$-valued L^2-functions on B or $L^2(B,\mu)\otimes\Lambda(H_R^*)^c$, a tensor product of the Bosonic Fock space and the Fermionic Fock space. On this space we define an exterior derivative d_A using D_A. Then $\frac{1}{2}(d_A^*d_A+d_Ad_A^*)$ $=d\Gamma(A\oplus\bar{A})\oplus d\Lambda(A\oplus\bar{A})$, a free Hamiltonian for an $N=2$ supersymmetric quantum field.

As in the finite dimensional case, d_A is decomposed as $d_A=\partial_A+\bar{\partial}_A$, and Laplace-Beltrami operators $\square_A$ and $\bar{\square}_A$ are defined as $\square_A=\partial_A^*\partial_A+\partial_A\partial_A^*$ and $\bar{\square}_A=\bar{\partial}_A^*\bar{\partial}_A+\bar{\partial}_A\bar{\partial}_A^*$, respectively. Since $\bar{\partial}_A^2=0$, $\bar{\partial}_A$ defines an elliptic complex and $\bar{\partial}_A$-cohomology groups can be defined as $\mathfrak{H}_A^{p,q}(B)=\mathrm{Ker}(\bar{\partial}_A|\Lambda_2^{p,q}(B))/\mathrm{Im}(\bar{\partial}_A|\Lambda_2^{p,q-1}(B))$, where $\Lambda_2^{p,q}(B)=L^2(B,\mu\colon\Lambda^{p,q}(H_R^*)^c)$, the space of square in-