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SPECTRAL PROPERTIES OF DIFFERENTIAL OPERATORS RELATED TO STOCHASTIC OSCILLATORY INTEGRALS

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1. Introduction

Recently, in the case of $\boldsymbol{R}^d$ with the standard flat metric, Schrödinger operator with magnetic fields has been studied by many authors, e.g., B. Simon and A. Iwatsuka. The semi-group generated by this operator can be represented in terms of stochastic oscillatory integrals (see [12] and [20]). Hence probabilistic methods play an important role in the study of this operator and its spectral properties are closely related to results in the stochastic analysis. The purpose of this paper is to study the spectral properties of Schrödinger operator of magnetic fields on $\boldsymbol{R}^2$ with a rotationally invariant Riemannian metric g. It is well known that if g is the standard flat metric, then Schrödinger operator of magnetic fields can have a wide variety of spectral porperties, (see, for example, [3]). In our case, the above variety still remains. In fact, we will show several similar facts to the results obtained by K. Miller-B. Simon, [14], A. Iwatsuka [8],[9],[10], etc.

Let M be a complete Riemannian manifold with a Riemannian metric g, α be a real valued differential 1-form on M and Δ be the Laplace-Beltrami operator on M, We consider a differential operator $L(\alpha)$ on M with the domain $C_0^\infty(M)^C$ defined by

$$L(\alpha)f = -\frac{1}{2}(\Delta f + 2\sqrt{-1}\langle df, \alpha\rangle - (\sqrt{-1}\delta\alpha + \|\alpha\|^2)f)$$

where $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$ denote the Hermitian inner product and the norm in complexified cotangent bundle $T^*(M)^C$ which are defined from g respectively, and $C_0^\infty(M)^C$ is the space of all complex valued C^∞-functions with compact support in M. In the case of $M=\boldsymbol{R}^d$ with the standard flat metric g, the operator $L(\alpha)$ is usually called the Schrödinger operator with the magnetic field $d\alpha$ (see [3]). Under some mild conditions, $L(\alpha)$ is essentially self-adjoint on $C_0^\infty(M)^C$ (see Section 3). Then $L(\alpha)$ can be uniquely extended to a self-adjoint operator $H(\alpha)$ on $L_C^2(M; dm)$, the Hilbert space of all complex valued functions on M which are square integrable with respect to the Riemannian volume dm. We