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THE ESSENTIAL SELF-ADJOINTNESS OF PSEUDODIFFERENTIAL OPERATORS ASSOCIATED WITH NON-ELLIPTIC WEYL SYMBOLS WITH LARGE POTENTIALS

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1. Introduction

In this paper we consider the problem whether pseudodifferential operators associated with real Weyl symbols are essentially self-adjoint on $L^2(\boldsymbol{R}^n)$. There is a great deal of work concerning this problem, especially for Schrödinger operators. (Seen Kato [14], [16, V, 5], [17], [18], Simon [23], Nagase-Umeda [20], T. Ichinose [8], and Iwatsuka [13] and the papers cited therein for example.) Most of them treated elliptic symbols having positive potentials with little regularity. Here we limit our consideration to smooth symbols only, but we do not assume the ellipticity of symbols or the positivity of potentials. Our aim is to give a simple sufficient condition on the growth of potentials for the essential self-adjointness. Besides, we give a counter example which shows the sharpness of our condition.

As an application of the above result, we obtain the $L^2(\boldsymbol{R}^n)$ well-posedness of the Cauchy problem for evolution equations whose evolution operators are time-dependent pseudodifferential operators associated with pure imaginary valued symbols, which include dispersive partial differential equations. Here and in the following non-Kowalevskian non-parabolic partial differential equations of evolution are called dispersive.

By a similar argument, we can show that the above Cauchy problem is well-posed on a family of weighted Sobolev spaces introduced by Beals [1] as well. Petrovskii [22] investigated the well-posedness of these equations with coefficients depending only on the time variable. Volevich [28] and Gindikin [5] generalized the above result to the equations with small potentials. W. Ichinose [9], [10] and Takeuchi [24], [25] studied the $H^\infty(\boldsymbol{R}^n)$ well-posedness of these equations with evolution operators associated with not necessarily formally skew self-adjoint elliptic symbols, and obtained some necessary conditions and sufficient conditions on the growth of the self-adjoint part of the symbols along the classical orbits. They also investigated the $L^2(\boldsymbol{R}^n)$ well-posedness and the