SOME PROPERTIES OF MARKOV PROCESSES ASSOCIATED WITH TIME DEPENDENT DIRICHLET FORMS

Dedicated to Professor Takesi Watanabe on his 60th birthday

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1. Introduction

In the previous paper [8], we showed the existence of the space-time Markov processes associated with certain time dependent Dirichlet forms. The situation treated there can not be covered by either the theory of symmetric Dirichlet forms or the theory of coercive non-symmetric Dirichlet foms. Nevertheless, as we saw in that paper, the methods of symmetric Dirichlet forms had been effective for the construction of space-time Hunt processes associated with time dependent Dirichlet forms. The purpose of this paper is to show that many of the properties similar to the case of symmetric Dirichlet forms given by Fukushima [4] still hold in the time dependent cases. To give the more precise statements of the results, we shall introduce the notations which will be used in this paper. Let X be a locally compact separable metric space and m be an positive Radon measure on X such that Supp $[m]=X$. We shall suppose that we are given a family $E^{(\tau)}$ ($\tau \in R^1$) of Dirichlet forms on $H \equiv L^2(X; m)$ with common domain V, that is, we are given a Hilbert space $(V, \|\cdot\|_V)$ which is densely and continuously embedded in H and a family of bilinear forms $E^{(\tau)}(\varphi, \psi)$ on $V \times V$ satisfying the following conditions:

(E.1) For all $\varphi, \psi \in V$, $E^{(\tau)}(\varphi, \psi)$ is a measurable function of $\tau \in R^1$.

(E.2) For any $p > 0$, there exists a positive constant $M = M(p)$ such that

$$|E_p^{(\tau)}(\varphi, \psi)| \leq M \|\varphi\|_V \|\psi\|_V ,$$

for all $\varphi, \psi \in V$ and $\tau \in R^1$, where by using the inner product $(\cdot, \cdot)_H$ in H, $E_p^{(\tau)}$ is given by

$$E_p^{(\tau)}(\varphi, \psi) = E^{(\tau)}(\varphi, \psi) + p(\varphi, \psi)_H .$$