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LARGE DEVIATION FOR A CLASS OF CURRENT-VALUED PROCESSES

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1. Introduction

In this paper, we are concerned with the large deviation problem for two typical current-valued processes among those that are induced by random curves: one is induced by Brownian motion, the other is by geodesic flow. For both processes, the law of large numbers, the central limit theoremes have been studied and there are some studies discussed relations between asymptotic behaviours of Brownian motion and geodesic flow (see e.g., Ledrappier [4]). These results suggest that the deviation funccions for two current-valued processes may coincide or at least have some connections, but since at present this remains unclear, we content ourselves to determine the deviation functions for those two currect-valued processes. Let M be a compact Riemannian manifold. We denote by $\Lambda^1(M)$ and $\Lambda^1(M)'$ be the smooth 1-forms on M and the currents, respectively. We denote by $(\mathcal{D}_1)_p$ the completion of $\Lambda^1(M)/\mathrm{Ker}\|\cdot\|_p$ with respect to the norm $\| \ \|_p$ (see, e.g., [5])., The dual spece of $(\mathcal{D}_1)_p$ is denoted by $(\mathcal{D}_1)'_p$. For a $\Lambda^1(M)'$-valued process $Y=(Y_t)_{t\in T}$, where $T=[0,\infty)$ or $\boldsymbol{R}$, we define the following quantities: Given a family of probability measures $\{m_x\}_{x\in M}$,

$$\bar{\lambda}(\Gamma)=\limsup_{t\to\infty}\frac{1}{t}\log\sup_{x\in M} m_x\,[\frac{1}{t}Y(t)\in\Gamma]\,, \tag{1.1a}$$

$$\underline{\lambda}(\Gamma)=\liminf_{t\to\infty}\frac{1}{t}\log\inf_{x\in M} m_x\,[\frac{1}{t}Y(t)\in\Gamma]\,, \tag{1.1b}$$

for any Borel set Γ in $(\mathcal{D}_1)'_p$ and

$$\Lambda[\alpha]=\lim_{t\to\infty}\frac{1}{t}\log\sup_{x\in M} E^{m_x}[e^{\langle Y(t),\alpha\rangle}]\,. \tag{1.2}$$

We call a function k an upper [resp. a lower] deviation function if

$$\underline{\lambda}(\Gamma)\geq -\inf\{k(\xi);\,\xi\in(\overset{\circ}{\Gamma})\} \tag{1.3a}$$

$$[\text{resp. } \bar{\lambda}(\Gamma)\leq -\inf\{k(\xi);\,\xi\in(\bar{\Gamma})\}]\,. \tag{1.3b}$$

In particular, we call simply k a deviation function if it is lower semi-continuous